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GENERAL PRINCIPLES OF PHOTOGRAPHIC SURVEYING

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[Communicated by Comdr. T. Y. BAKER, R.N.]

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ABSTRACT. The paper gives a brief outline of the development of photogrammetry from terrestrial and aerial positions, and an indication of some of the points to be considered in the design of instruments required by the photogrammetric surveyor in this country.

SIMPLE PHOTOTOPOGRAPHY

IN its early stages photographic surveying was confined to the use of a camera as a record of angular measurement for graphic use. It was unusual to take careful measurements from the plate for the purpose of subsequent compilation. It was common to plot at once on paper, and the accuracy aimed at was thus only that possible to direct plotting—i.e. something of the order of 0·005 inch. Since the angle allowed for intersections was generally not under 20° the resulting positions were of the same order as those obtained by plane-tabling, so long as the points fixed were clear, sharply defined, and properly identified.

The principal distance was generally established by photographing points or marks whose angular distances at the point of exposure were known, and the principal and horizontal lines were found by photographing the horizon in different positions of the camera.

Surveys of this type presuppose mountainous country with features recognizable at any angle, and have had, therefore, no vogue in England and little in the British Empire except in the Rockies. Fig. 1, which is taken from a paper by M. P. Bridgeland*, is a representative survey photograph of this type and shows the kind of country to which the method is applicable.

* M. P. Bridgeland, "Photographic Surveying," *Topographical Survey of Canada, Bulletin No.* 56 (Department of the Interior, Canada).

The camera used in Canadian surveys is a typical, strong, simple, and effective one of its kind; its construction can be seen from Fig. 2, which is taken from the paper just mentioned. It weighs 20 lbs. complete in case with twelve loaded plate-holders. The lens is a Zeiss Tessar, Series III, the focal length of which is 164 mm. The horizontal and vertical fields are 51° and 37° respectively.

The plates used are Wratten and Wainwright's panchromatic, and a yellow screen is employed to lessen the evils of "aerial perspective."

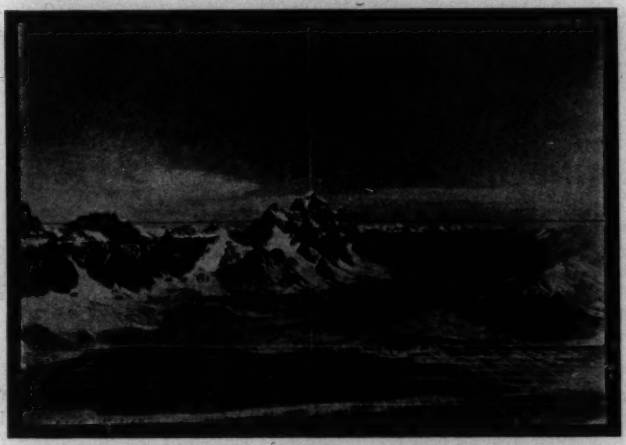


Fig. 1

It should be mentioned that a short exposure is of no particular importance (in contradistinction to air photography) and that the lens can therefore be stopped down.

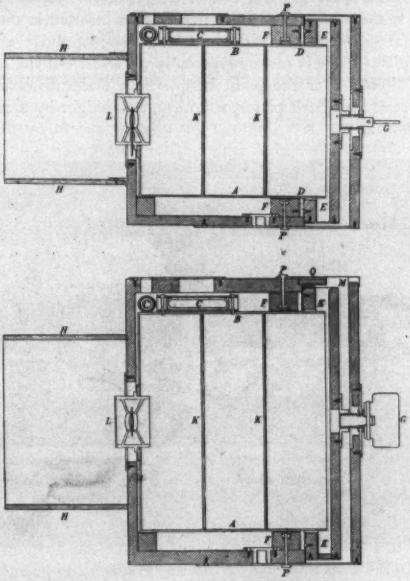
It should also be mentioned that simple interchangeable cameras and theodolites have held the field in Canada in preference to the combinations called photo-theodolites produced for alpine surveys.

Non-automatic Recording Instruments

The greatest drawback to simple phototopography is the difficulty of identifying the same point from different directions; often difficult in very broken country, it becomes impossible in a rounded undulating landscape. The plane-tabler's rangefinding sense, stereoscopic in the foreground, and dictated by colour, tone, aerial perspective, and comparative size in the distance, enables him to

place an object fairly correctly on a single ray. Identification is not difficult. On a photograph most of these factors are lost.

In 1908 Capt. F. V. Thompson, R.E., read a paper to the Royal Geographical Society on the application of stereoscopy to photogrammetric plotting*. Two or



AB Metal box open at one end.
C, C Two sets of cross levels.
D, D Bolts.
EE Carrier for plate-holder.
FF Frame for holding *AB*.
G Screw for moving *EE*.
HH Folding shade.
K, K Diaphragms.
L Lens.
M Opening for insertion of plate-holder.
P, P Bolts.
Q Clip.

Fig 2. Cross-sections of Canadian Camera

three models of a plotter made to his design are still extant. Dr E. Deville† of Canada was also a pioneer in the theory, but pursued the matter no further on learning that Prof. Pulfrich of Jena was ready with a practical solution.

The stereocomparator (Fig. 3) produced at Jena in 1902 by Prof. Pulfrich‡

* F. V. Thompson, *Geograph. Journ.* 31 (1908) 534.
† E. Deville, *Trans. Roy. Soc. Canada* (1902–3) p. 63.
‡ C. Pulfrich, *Z. f. Instrk.* 22 (1902) 65.

enlarges the eyebase to the length of a small measured base (1000 yards or so). Very small angles of intersection are not only general but indeed necessary so as to keep within the general limits of stereoscopic fusion. Exposure is limited to parallel lines in the horizontal plane. Distance is measured by the angle at which rays from the eyes converge, and the convergence is affected by moving one plate relatively to the other. A "survey mark" (one inserted in the optical system for each eye) is permanently and stereoscopically "fused" and the relative motion of the plates make it move, apparently, to and from the observer until it appears to be directly over the point in question. Azimuth is obtained by the combined movement (of both plates) sideways, and height by a combined movement up and down.

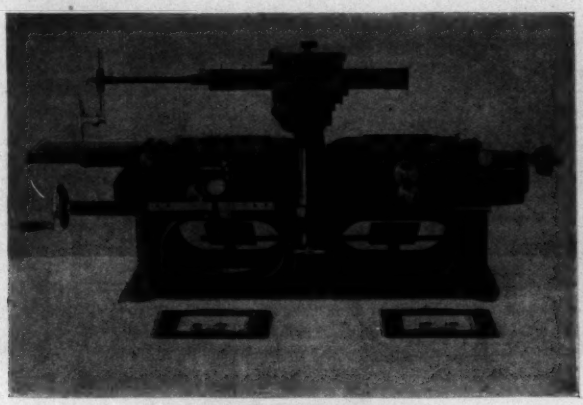


Fig. 3. Pulfrich Stereocomparator

The surveyor is becoming more exacting, for he must have for this method

(*a*) a precise base measurement, preferably optical,
(*b*) a more precise camera,
(*c*) better plates.

The telemeter and phototheodolite of Wild (of Heerbrugg in Switzerland) are probably the best on the market.

Automatic Recording Instruments

Widely used as the comparators have been (F. V. Thompson's was used for the survey of Fiji and Zeiss's comparator is heard of so far afield as the Taurus Mountains and Pekin), plotting was effected point by point and mistakes arose unless measurements were checked.

The next stage therefore is the von Orel stereoautograph (Fig. 4) perfected by the Austrian officer of that name in connexion with Zeiss of Jena. Here we have the movements of the comparator carried by levers through a geometrical system to a pencil which plots directly upon paper. No errors of recording or transcribing are possible here, and by clamping the vertical movement the pencil can be made to trace out contours as accurately as the eye can "fuse" simultaneously survey mark and landscape. No details of this instrument need be mentioned here. It has been well and thoroughly described in the *Mitteilungen des k. u. k. Militärgeographischen Institutes, Wien* (1911) and a description has also appeared in the *Journal of the Royal Geographical Society*, 59 (1922) 273.

But this instrument has now, in my opinion, been much improved upon by Wild of Heerbrugg whose stereocartograph, comparatively minute in size, is a marvel of elegant instrumental design. The design is radically different from that of von Orel and allows of axes of exposure at considerable angles (in any direction) to each other.

The surveyor is now using an instrument calling for the highest order of design based on geometrical optics, and lenses which must be precisely similar in camera and plotter.

As a result of this last stage, photogrammetric surveying has been made possible in nearly every type of country, a formidable rival of any other survey method, and claims pre-eminence in mountainous country at medium scales (1/5000 to 1/40000).

Aerial Photogrammetry

The development of surveying from air photographs is taking a course very similar to that of ground photographic surveying. The first use of air photographs and the commonest to-day is to secure a plan by photographing on as nearly a vertical axis as the navigation of the plane will allow, and to join up individual prints to form a mosaic pinned down here and there on the earth's surface on fixed points. One cannot measure relative height or contour from such mosaics of course, but one can arrive quickly, and without any particular skill, at the planimetry. For this procedure little need be known of the lens so long as it covers the plate fairly well. In fact the one preoccupation is to secure a good clear image.

Still analogous to the first single plate stage of ground phototopography are the perspective methods of "rectifying," or plotting from, oblique views. The laws of perspective which govern these matters will be dealt with by Col. MacLeod in his paper*.

Meanwhile it must be pointed out that for these developments we must have very precise knowledge of the interior constants of the camera. Indeed it is obvious, and will be more obvious later from Lt. Hotine's paper†, that an accurate calibration of lens and camera, and good photographic plates are the first essentials to progress.

Perhaps the most important and difficult question before us may, however, now be mentioned. In ground work one can easily and precisely obtain the exact

* See p. 74. † See p. 84.

Fig. 4. von Orel and Zeiss Stereoautograph

position of the camera, but in the air this is extraordinarily difficult; it cannot be done directly in a body moving at a speed of 100 miles an hour or so. Even the height, which might be presumed to remain fairly constant, cannot be measured at present closer than 100 feet or so.

The survey has to be done indirectly from the images on the plate of a number of surveyed positions on the ground. Now the position of the camera has, relatively to the ground, three degrees of freedom. The plate may be considered subject to three further degrees of freedom. The images may be poor, and are seldom really good. The "pyramid," whose apex is the camera and whose base is the triangle or quadrilateral on the ground, is generally tall and "acute." But

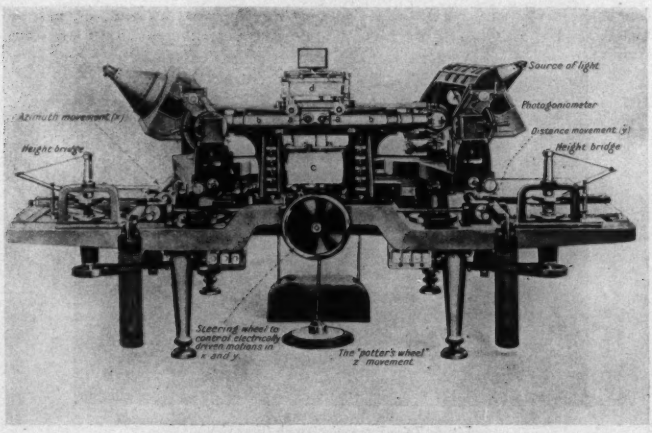


Fig. 5. Hugershoff (Gustav Heyde) Autocartograph

this problem must be solved—and for precise work it must be solved precisely. We may do it algebraically, but a single solution may take a day. We may do it by trial and error, but the error predominates. It is interesting to note a recent experiment in France where the position was worked out mathematically and compared with that obtained by the simultaneous observation of three theodolites. There was a discrepancy of 200 metres. We are hoping to evolve an optical instrument which will allow us to obtain a sensibly accurate solution, but we shall never obtain a precision comparable to that measurable on the ground.

Exactly parallel to the first stereoscopic development in ground photographic surveying (the Zeiss comparator) is the Brock system in the U.S.A. The principle of the system is that of a comparator used to find the actual distance of the base

from a point which should subtend a definite parallactic angle. It is, in fact, like reversing a computation. When you have found your "base" (and incidentally surveyed its ends) you can return to the direct function again and survey other points. There is a good deal to be said for the Brock system, but it includes a somewhat elaborate programme of tilt-finding, rectification, plotting, and contouring.

The final development of stereoscopic and direct plotting from ground photographs (the von Orel stereoautograph and the Wild stereocartograph) has already been copied by the autocartograph (Fig. 5) of Hugershoff (Gustav Heyde) and the stereoplanigraph (Fig. 6) of Zeiss*.

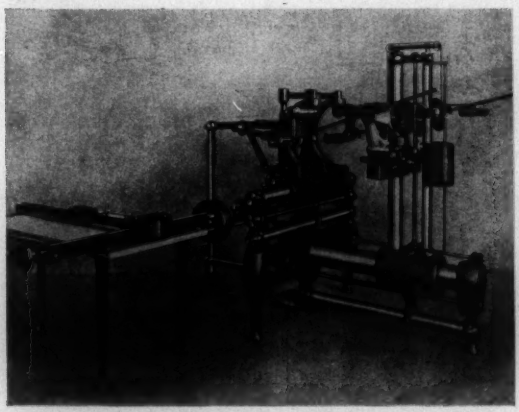


Fig. 6. Zeiss Stereoplanigraph (near view)

General Remarks

Are we to assume then that the question is solved—that it is, as the French say, "au point"? Yes and no. Maps can be made with these instruments, but the survey of the camera position is still cumbersome. They will both act in the hands of those who built them, but I have reason to suspect do not do so out of them. It is doubtful if the construction of one of them is altogether free from reproach, and it is very doubtful if, for anything but the most difficult country, their use will repay their cost.

French models and an Italian one (on a different principle) are being con-

* See O. v. Gruber, *Z. f. Instrk.* 43 (1923) 1.

structed. In fact we are all seeking the solution on these lines, for we believe that, as ground photographic surveying has ended in the stereoautograph, so must air phototopography ultimately be carried out with the aid of an instrument resembling, say, the autocartograph (but one that is lighter, smaller, and easier to handle), preceded by a quick and accurate measurement of the position in space and the "set" of the plate.

I have tried to outline the development of photogrammetry from terrestrial and aerial positions and to show how that development has followed the same sequence. We now want our British tilt-finder and our British stereoscopic plotter. The designing of such instruments involves exercises in geometrical optics which I am sure are worthy of your attention.

Figs. 3 and 4 are reproduced from *The Geographical Journal*, 59 (April, 1922) with the kind permission of the Royal Geographical Society. The original photographs were lent to that Society by Colonel Talon.

For discussion see pp. 89–93.

PERSPECTIVE CONDITIONS OF PHOTOGRAMMETRY

By Lt.-Colonel M. N. MACLEOD, D.S.O., M.C., R.E.

[Communicated by Comdr. T. Y. Baker, R.N.]

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ABSTRACT. The perspective conditions which must be satisfied in aerial survey work are discussed and the methods of reconstructing these conditions from the photographs obtained are described. Attention is drawn to the desirability of designing new apparatus with which to carry out the necessary reconstruction with the requisite accuracy.

Use of the Camera

The principles governing the employment of the camera as an instrument of survey are based on the definite assumption that the image formed on the plate is a true perspective view of the ground. That is to say, the positions of any elements of the landscape on the plate are defined by the intersection with the plane of the plate of straight lines, drawn from the elements on the ground, through a single point in, or in the vicinity of, the lens.

In striving after perfection of the camera for survey work, the designer has to satisfy this condition as far as it is possible to do so without sacrifice of definition. Failure to satisfy perspective conditions introduces errors due to the irregular displacement of points on the plate; lack of definition equally will introduce errors due to the impossibility of identifying with certainty the position of a point or object whose outlines are not clearly defined.

One must remember that in surveying with a theodolite or plane-table one is making on the ground, at full size, measurements which are afterwards plotted on the map at a very greatly reduced scale. In surveying with the camera, our measurements have to be made on a photographic plate which, in many cases, may be of a scale smaller than that of the map. It is quite obvious that, if comparable results are to be obtained in each case, we shall require from the camera itself very perfect definition, and that not only must the mechanism used for making the measurements be very refined, but also the data with which this mechanism deals must be determined in a very precise manner.

Regarded as an instrument of survey, the camera is used in two quite distinct ways. In the one it is used in place of the theodolite or alidade to measure angles or directions, this use being typical of photographic surveys executed on the ground; in the other it is used to obtain directly a complete plan or perspective view of the ground, this use being characteristic of photographic surveys executed from the air. These two uses, though essentially quite distinct, can be combined,

the one supplementing the other. The data required for each are the same, but are used in different ways.

To measure directions and angles from a photographic plate, we may start by measuring linear distances on the plate from the plate centre to particular points and from these distances compute the angles subtended at the back node of the lens. For this computation we must know the position of the optical centre of the plate and the principal distance. Furthermore, since for survey purposes we require our angles always to be recorded in either horizontal or vertical planes, we must know the direction of the horizontal on the plate when the plate was taken.

The Photogoniometer

Alternatively, we may employ the photogoniometer (Fig. 1), replace the photograph in the camera (or a replica of the camera) and look at it through the lens with a telescope mounted in a manner analogous to that of a theodolite telescope, so that its centre of rotation coincides with the centre of the camera lens.

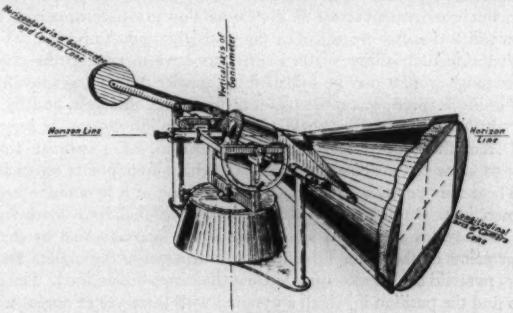


Fig. 1

To use this apparatus one must know the direction of the horizon on the plate, the optical centre, and the principal distance. I say principal distance rather than focal length advisedly; it is desirable, at any rate in theory, that the two things should be identical, but, practically, it may not be possible to obtain exact agreement. The focal length is a function of the lens and determinable from it without reference to the particular camera in which the lens is mounted. The principal distance is a function of the camera and is the perpendicular distance from the rear nodal point of the lens to the plate. One may say that the geometrical conditions of photogrammetrical perspective are based on the principal distance. If this differs from the focal length, the geometry is unaffected; all that happens is that definition may be bad and the image blurred. Of the two, it is less important to get perfect definition than it is to ensure exact determination and preservation of the principal distance.

Conditions of Photographic Survey

All this is perhaps rather elementary. The camera has been used for many years to measure angles and its attributes in this respect are well known. I will therefore devote no more time to this side of the subject but turn to the consideration of surveys in which the photograph is used to give a direct plan. It may be assumed that the camera is carried in an aeroplane and is moving at a speed of anything up to 100 miles an hour. The photographs themselves may be direct vertical views, or they may be purposely tilted at an angle of anything up to 45° from the vertical.

Two elementary necessities in such a system are that the method of exposing the plate should be sufficiently rapid and that by some means or other the position of the camera at each exposure can be determined. As to the method of exposure one must postulate some sort of shutter which does not allow the position of the perspective centre to change appreciably while the exposure is being made. A focal plane shutter can be made rapid enough over any part of the plate to give sharp definition, but its movement across the plate is too slow to satisfy the perspective conditions on which the after-treatment of the photograph must necessarily be based.

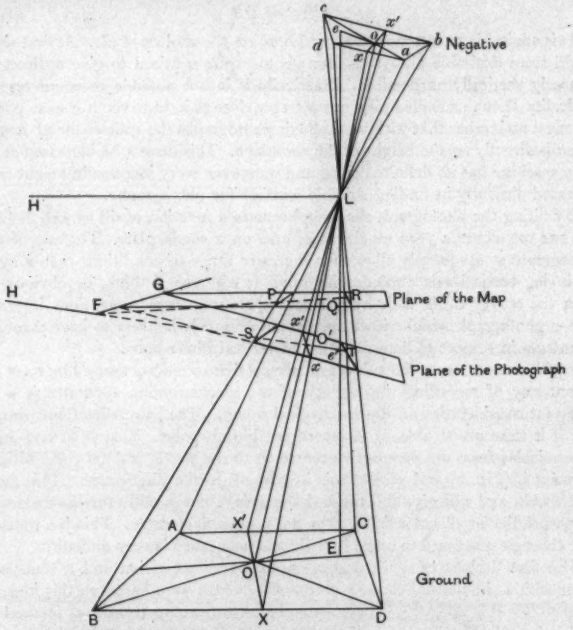
For deducing the position of the camera we have to rely on the photograph itself. For rough work it may be sufficient to organize the photography in straight flights at uniform speed, to expose plates at fixed time intervals, and by locating only two or three points in the course of the flight, to interpolate the intermediate positions from the time interval between exposures. For precise work it is necessary to know the positions of three or perhaps four points on each photograph. These four points we may regard as the base of a pyramid whose vertex is the camera lens. On the opposite side of the lens we shall have another pyramid with the same vertex angles but whose base is the figure formed by the images of the four points on the plate. Provided the dimensions of the camera are known, the camera pyramid can be reconstructed and the vertex determined. The problem is then to find the position in which a pyramid with these vertex angles will fit on to the basal figure.

Reconstruction of Perspective Conditions

In general, aerial photographic survey methods are based on a reconstruction of the perspective conditions such as is implied in this description. Three methods of solution are available which I may call mathematical, graphical, and mechanical. The first two are satisfactory enough when the control triangle or quadrilateral is a plane figure in the datum plane, i.e. if the control points are all of the same height. When this is not the case both become complicated and difficult.

The mechanical solution depends on a complete reconstruction of the perspective conditions expressed by the two pyramids referred to. The control points are plotted on an adjustable screen representing the datum plane and due allowance is made for their heights. The photograph is then projected on to this screen in a way which parallels the perspective conditions in the aerial camera.

If the screen can be manipulated until the projected positions of the control points coincide with the plotted positions, the problem is solved and it only remains to record the positions of the various parts of the apparatus thus found in a convenient and suitable manner.



xx' represents the horizontal line through the centre *O* of the Negative.
eLE represents the vertical line through the Lens *L*.

Fig. 2

This reconstruction, if it can be achieved, does more than simply fix the position in space of the camera. Colonel Winterbotham has mentioned three degrees of freedom of the camera and three degrees of freedom of the plate*. Let us see what these are. Fig. 2 shows the geometry of the conception. The camera position may be defined by three rectangular co-ordinates, two horizontal and one vertical. These are the co-ordinates of the plumb point and the height. In respect

* See p. 71.

to the plate we have to find the angle of tilt and its direction with reference to a fixed and arbitrary line on the photograph and with reference also to the co-ordinate axes which define the camera position. These two last items are referred to as the "swing" and the bearing of the principal plane.

Effect of Tilt

It is advisable to interpolate a word here on the subject of tilt. At first sight it would seem desirable always to take photographs required to give a direct plan as nearly vertically as possible. Practically it is not possible to count on exact verticality, though a skilled pilot can get very close to it, but even if it were possible one must remember that with vertical axis photographs the scale of the photograph depends directly on the height of the aeroplane. This cannot be increased at will; every machine has its definite ceiling and moreover every increase in height entails increased difficulty in finding suitable weather for photography.

By tilting the photograph the scale becomes a function of tilt as well as height and one can cover a very much larger area on a single plate. The use of tilted photographs consequently allows for a greater range of conditions than a system employing vertical-axis photographs only. It will also, I think, be obvious that, when the survey deals with heights and contours—three dimensions instead of two—a photograph which views the ground obliquely is likely to have theoretical advantages in respect to determination of vertical dimensions.

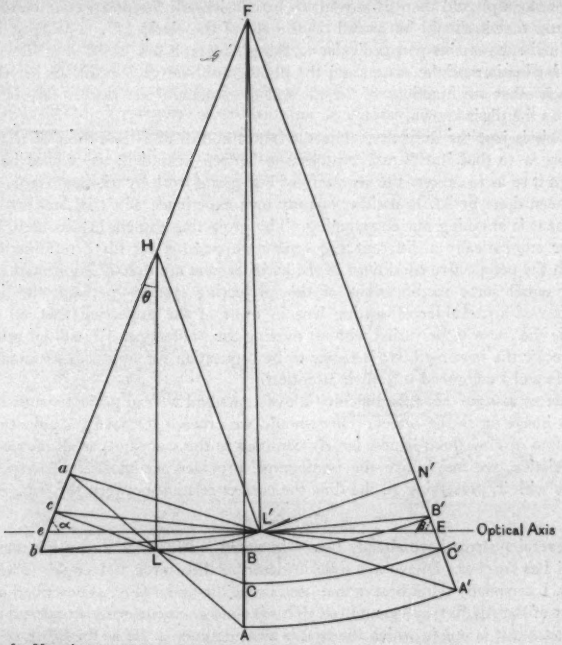
I will now return to the subject of perspective reconstruction. The most convenient way of recording the settings of the reconstruction apparatus is a very important consideration in the mechanical design. The most direct and simplest way, if it were practicable, is to record the plumb point. That is to say, drop a perpendicular from the perspective centre on to the screen and get the position of the point E (Fig. 2) and project this in turn on to the photograph. This part is not difficult, and will give the tilt and the swing, but it still remains to measure the perpendicular distance from E to the perspective centre. This is a measurement through space and to make it easily and accurately is very difficult.

The first designs of a tilt-finder employing direct vision and a transparent screen with a simple pinhole as a perspective centre were based on this idea, but owing to just this difficulty it now seems better to give up this direct method and to record the inclinations of both photograph and screen about definite axes and the distance apart of these axes and compute all the elements required from these data. It is regrettable that one has to introduce computation even though it is not a particularly difficult computation, but it seems to be the best solution.

Method of Projection

Another very important consideration is the method of the adjustment for perspective coincidence. This is by no means an easy matter and its facility depends in great measure on the method of projection. The obvious method of projection is a beam of light. The ordinary projecting lantern has been and is still used, but its great drawback is that, to make the coincidence accurately, one

requires a sharp image on the screen, and to get a sharp image one requires freedom of movement for the photograph as well as for the screen. The aerial camera is



ab = Negative.
AB = The plane of the Map under the original conditions in the Aeroplane.
L = The Lens of the Aeroplane Camera.
L' = The Lens of the Enlarging Lantern.
$eL'E$ = The Optical Axis of the Enlarging Lantern.
$A'B'$ = The Copying-board set in position.
c = The Centre of the Negative. $eL' = d$, $EL' = d'$, $ec = x$.
H = The intersection of a horizontal line through the original perspective centre with the plane of the Photograph.
i.e. LH is parallel to AB also $L'H$ is parallel to $A'B'$.
F = The intersection of the planes of the Photograph and Map.
The Negative and Copying-board pivot about axes at e and E perpendicular to the plane of the paper.
The horizontal line through c (xx' in Fig. 2) must be placed parallel to these axes.

Fig. 3

focussed for infinity; naturally we cannot just reverse it and expect a sharp image on a screen at a distance of 2 feet. Now freedom to move the photograph is just

what we have not got. Satisfaction of the perspective conditions imposes strict limits. These limits are seen from Fig. 3. If we vary the distance of the lens from the photograph and there is any tilt to be considered, the perspective conditions demand that it should be moved on the arc of the circle LL'. The optical axis $eL'E$ neither remains perpendicular to the photograph nor meets it at its centre. The inclinations of the screen and the photograph and their positions in relation to each other are functions of the tilt and the scale and are readily calculated as soon as the tilt is known.

This is just the difficulty. The tilt is not known and the object of the projection is to find it. To use a projecting lantern to find it, we should have to design it so as to express the geometry of Fig. 3 and work by trial and error. This has been done by M. Roussilhe, but my own experience of a trial and error adjustment is anything but encouraging. The projecting camera has its uses, but I prefer emphatically a different and separate apparatus for tilt determination in which the perspective conditions of the aerial camera are exactly duplicated. This must entail some modifications of the projecting system—perhaps the introduction of a movable collimating lens in front of the projecting lens, so as to enable the focus to be varied without moving the photograph. I will not attempt to specify the best method; it seems to be a question for optical instrument designers and I commend it to their attention.

Let us assume that this difficulty is overcome and we can project a sufficiently sharp image on to the screen. How should we proceed to make the adjustment? Freedom of movement is now largely confined to the map, but, as all movements are relative, we may move the photograph provided we move the perspective centre with it, preserving all the time the correct relation between the two.

Tilt-finder

Several systems have already been "invented": M. Predhumeau has one and Nistri has another. I have not time to describe them even if I could. The one which I personally think best is that used in the tilt-finder (Fig. 4) described in the report of the Air Survey Committee. This is an instrument using a trial and error method but it is one in which the trial is systematic in so far as the effect of each movement on the perspective is readily seen; after very little practice one can see in making the trials whether the movements are being made in the right direction or not. This is not the case with all the others. In some it is possible to see (though not, I think, so readily) whether the trial movements are or are not in the right direction, but with others this is not the case and the trials are purely haphazard. A purely haphazard system is not satisfactory and may lead to hours of time wasted without attaining any result in the end.

Rectification of Photograph

Let us now pass on from the subject of determination of plate constants and other data to the actual use of the photograph. Our material is a perspective view taken under conditions which are now adequately known. What are we to do with

it? Our object in essentials is to get an undistorted plan which can be traced direct on to the map which it is our aim to produce, and the next step must be to get rid of the perspective distortion by "rectifying" the photograph. For this we may revert to the projecting camera and project the photograph anew but under different conditions in that we know the tilt and factors which affect the projection. We have in consequence much greater freedom in the arrangement of our data; we need not attempt to duplicate exactly the perspective conditions of the aerial camera. We require sharp definition as before, but we may, if it is convenient, move the photograph as well as the screen, provided our movements are in conformity with certain principles which can be seen from Fig. 3.

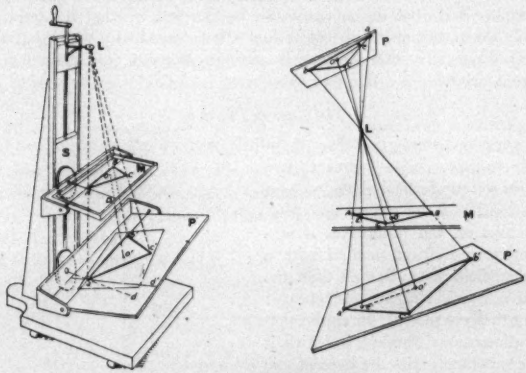


Fig. 4. Tilt-finder (Mark I)

The perspective principles which have to be followed are that, the principal plane having been made to coincide with the plane of the figure, the lens must lie on a circle whose centre is the "horizon trace" H on the photograph and the plane of projection must be parallel to the plane containing the lens and the horizon trace and must pass through the perspective axis F. The optical condition governing the projection is that the optical axis must be perpendicular to the plane containing the lens and the perspective axis. These conditions are easily expressed as a series of equations from which the positions of the different parts of the apparatus may be computed.

It may be desirable to specify here the particular movements required by a rectifying camera to enable the results of these equations to be expressed. The movements required are

(*a*) a photograph holder which enables the photograph to be centered accurately and rotated in its own plane as well as rotated about a horizontal axis,

(*b*) a screen which can be rotated about a horizontal axis, parallel to the axis of rotation of the photograph,

(*c*) a longitudinal slide carrying the photograph, lens, and screen, so graduated that the distances from each other are easily measured,

(*d*) a lens which can be rotated or "canted" about a horizontal axis parallel to the axes of the screen and photograph.

The theory of rectification is based on certain assumptions which, though commonly made in optical practice, are not strictly justified. One assumes, for example, that the lens has a flat field of indefinite extent. To rectify a photograph taken at a tilt of 30° might entail inclinations of negative and screen at angles of 45° to the optical axis. Having never attempted to rectify a photograph such as this, I cannot say what the image would be like and whether it would remain reasonably sharp, but up to 10° tilt at least—the greatest with which I personally have worked—there is little difficulty in getting reasonably good definition and an undistorted print.

The Camera Plastica

It is curious, I think, that there is so little practical experience of rectification. It seems to have fallen between two stools—the strictly vertical photograph on the one hand and the stereoscopic plotting machine on the other—yet it is a very simple and direct method with a much wider field of application than the first and a much simpler and cheaper technique than the second. Its application to three-dimensional surveys, in which it will have to compete with the stereoscopic plotting machine, is exemplified in the "camera plastica," a device for projecting two photographs on to the same screen. The effect of accidented ground on the photographic perspective can be seen from Fig. 5. If two such photographs are projected on to a screen representing the datum plane, two images M_1 and M_2 of the point M will appear. If the screen is gradually raised, these images will approach each other and finally coincide when the screen has been raised through a distance representing the height of M at the scale of plotting. The position of the image on the screen in this position is clearly the map position of the point.

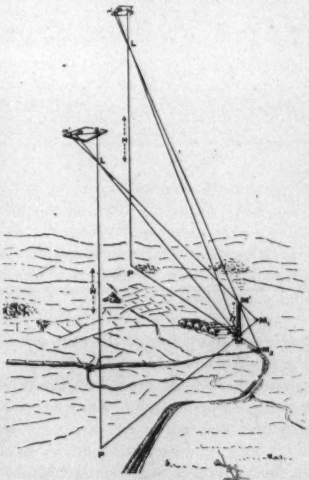


Fig. 5. Displacement of photograph due to height of feature

This principle would appear to be a good deal simpler than that of the stereoplotter, but its application is hampered always by the need for keeping sharp definition. For instance, if the photographs are taken from different heights, a screen distance which will give sharp focus for one projecting camera will not suit the other. If the same type of projecting camera is used for each photograph, the scale of rectification for one is not right for the other and so on.

I cannot but think that the relative simplicity of the apparatus required for this method makes it one which has very considerable possibilities. The difficulties arise solely from the system of projection which one is compelled to use. Could one eliminate a lens from this or design a lens with focal length easily varied, without infringing perspective conditions, I feel sure it could be turned to immediate practical use. I hardly like to suggest that it will be suitable for really precise work, but where contouring has to be done and where speed and cheapness, rather than precision, are the first factors for consideration, the double projecting system of the camera plastica seems to offer by far the simplest solution of the problem.

In all aerial photographic work this question of precision constantly recurs. Aerial survey is, and must always be I think, much more dependent on "apparatus" than ground work, and in the use and design of this apparatus the preservation of due proportion between the refinement and cost of the apparatus and the work required of it is a matter of prime importance. This question is dealt with in Lt. Hotine's paper*.

For discussion see pp. 89–93.

* See p. 84.

SOME PRECISION PROBLEMS IN AIR SURVEY

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[Communicated by Comdr. T. Y. Baker, R.N.]

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ABSTRACT. The various factors upon which accuracy in air survey measurements depends are described and suggestions are made as to some of the conditions which should be fulfilled in apparatus employed for such measurements.

Required Degree of Precision

The degree of precision required in the construction of air survey apparatus depends on the scale and purpose of the finished survey, the characteristics of the ground surveyed, and the conditions of height, tilt, etc. under which the photographs are taken. It is quite impossible to lay down a standard of accuracy in general terms, and the only safe guide is to aim at as high a degree of precision in construction as possible, having due regard only to the practical considerations of weight and bulk. The capital and depreciation costs of the camera, for instance, will in any case be only a small fraction of the cost of the survey and need hardly be considered.

From the test data of this apparatus the surveyor may then estimate the degree of photographic and instrumental precision which he may expect, and by varying the conditions of photography reconcile this to some extent with the accuracy required in the finished map. If, for instance, he wishes to produce a precise large scale contoured map of gently undulating country, the inaccuracies in his camera may force him to sacrifice economy by taking low altitude obliques and plotting from restricted areas only of the photographs. If, on the other hand, the immediate object is an uncontoured map at a small scale, he may deliberately sacrifice photographic precision in favour of speed and economy by using, say, a wider angled lens.

There is, however, another aspect of the question. Force of circumstances may restrict the diversity of apparatus at his disposal or fix independently the conditions of photography. The required precision of the apparatus is then the precision of its most exacting application, and again we reach the conclusion that the required degree of accuracy is the highest obtainable.

Furthermore, the problem of survey from the air will not be confined in the future to the plotting of planimetry and relief from an individual pair of controlled

photographs. The provision of close ground control is relatively slow, expensive, and in some cases impossible; and to realize the full value of the method the camera must act as its own theodolite and provide its own secondary control. Now an uncorrected error of one hundredth of a millimetre anywhere on the plate is the greatest degree of linear precision which is ever likely to be attained in the measurement of an air photograph without undue complication. Under average conditions, in the type of apparatus contemplated for this process of photographic triangulation, this corresponds to a horizontal angle of some twenty seconds. Linear errors in the photograph, however, have a double effect; first, by giving rise to errors in the plate constants on which the subsequent calculation or plotting is based, and second, as a direct error in the measurement of the required control points. It would thus appear that the limiting accuracy in the determination of horizontal angles would be more likely thirty to forty seconds, and this assumes that the results are calculated and are not subject to further instrumental error in the plotting machine or photogoniometer. Photographic extension of triangulation to a useful order of accuracy may thus eventually be possible, but will require a very high standard of optical and mechanical construction. It must come, however. A greater degree of angular precision can of course be obtained by using a longer focal length and thereby increasing the weight and bulk of the apparatus in something like a cube ratio. Until the last iota of precision has been wrung from the construction of smaller cameras, however, we should be unwilling to sacrifice economy to this extent.

Causes of Want of Precision

I will now turn to the various causes of want of precision in an air photograph and attempt to give some indication of their relative importance.

Definition

Firstly, as regards lack of definition due to approximately *symmetrical* diffusion of the image, the contributory causes are the symmetrical aberrations of curvature and astigmatism (as far as one may separate, in practice, the theoretical aberrations of lenses), short period oscillations of the camera mounting and other causes of movement of the camera during the necessarily finite exposure, the grain size of the emulsion, light scatter, etc. The practical desirability of critical definition for the precise measurement of distances is obvious, although the actual errors due to these causes are found to be small. In a particular case, using a 10-inch Tessar objective which was uncorrected for the presence of a glass pressure plate placed in front of the negative, the probable error of a single observation due to these causes was found by direct measurement to vary from fifteen to twenty thousandths of a millimetre for different objects in different parts of the field. With a more suitable objective, such as the Cooke Aviar, there is little doubt that even these errors will be reduced.

The most pressing need for improvement in this field would seem to be the provision of a fine grained and yet fast plate. This question is, I believe, re-

ceiving the attention of the British Photographic Research Association. The practical considerations of weight, bulk, and the desirability of a wide angular field on a medium sized plate, all tend to decrease focal length. It is reasonable to suppose that errors due to these causes will be decreased in proportion, with, however, the sole exception of grain size, which is at present mainly a function of speed. It is likely, then, that the grain size of the emulsion will assume a greater importance in the future than it does at present.

The position of an image "point" as thus estimated is not its correct perspective position by reason, in the first case, of distortion and coma. Where precision is of primary importance the plotting will normally be done in a machine where the image is re-projected through the same or a similar lens, placed in the same position relative to the plate as was the camera objective. Errors due to distortion will not arise in this case. Moreover, in any process of tilt-finding, subsidiary to machine plotting, it will usually be possible to allow for such errors. Correction for distortion is thus of secondary importance, although in the wider applications of the subject it cannot be entirely neglected. In the case of the 10-inch Tessar objective mentioned above, coma would account for a mean error of the order of one-eighth of a millimetre, which would, moreover, vary with the position and brightness of the image and would be difficult to correct with any semblance of truth. The elimination of coma in objectives for photographic survey is thus of primary importance.

Photographic Plate or Film

A further source of error lies in the photographic plate or film. In the case of film, uncertain changes in the nitro-cellulose or aceto-cellulose base, in the gelatine emulsion, or in the relative slip between the two, result in a displacement of the image. As an indication, it has been found in one particular test that there is a mean displacement of the order of one-tenth of a millimetre relative to a mean scale. Moreover, this result is found to vary in different samples of the same film, or even in the same sample under different conditions, and is little indication of the maximum error to be feared. This possibility of reasonably large unsystematic error means that film cannot be used at present for work of the highest order of accuracy, although advantage can, and always will, be taken of its practical virtues for filling in detail or for revision purposes. There is a clear field for chemical rather than optical research in the production of "distortionless" film.

The corresponding errors in glass plates are practically negligible for our purposes, tests at the Royal Observatory, Greenwich, having shown that, in carefully dried plates, no greater linear error than three or four thousandths of a millimetre is to be feared. There is, however, another source of error in this case due to the not inconsiderable curvature of commercial plates. Small linear perspective errors are thus introduced in addition to a certain loss of definition. This curvature would seem to be due to the practice of cutting photographic plates from a "sausage," and also to the pull of the emulsion in drying. Fortunately the plates are invariably concave on the emulsion side, and can be flattened in the

camera and in the plotting machine, at the expense of a little extra complication, by a back pressure pad. From the point of view of both convenience and precision, however, it would be a clear advantage if a stiff and reasonably flat plate could be produced in the first instance. This can always be done by using thicker glass, but considerations of weight and bulk will not allow this until the aeroplane is built round the camera. Is there not some other way?

Camera Shutter

The camera shutter presents a difficult problem, by reason of the vibration and extreme temperature changes to which it is subject. Without going fully into the question, it may be assumed that between-lens shutters cannot be used at focal lengths of 10 inches or over, and are seldom satisfactory at even shorter focal lengths. The choice has consequently fallen on focal plane shutters. In this case, however, in addition to mechanical troubles there is a certain introduction of error owing to different parts of the same negative being in different perspectives—an error which has been found to have a mean value of about one-fifth of a millimetre in a particular case. It is hoped that these difficulties will be overcome in a new type of shutter now being evolved.

Filter

For general work in varying atmospheric conditions the lens must be furnished with a filter. Up to the present this has taken the form of a simple gelatine disc between the lens components—an unfortunately necessary arrangement which did much to destroy the best work of the lens designer. A subsequent proposal was to use a gelatine disc cemented between flats and supported in front of the lens. This would doubtless be an improvement but adds the climatic troubles of Canada balsam to those of gelatine. Messrs Chance have now, we hope, solved the problem both from the point of view of precision and of permanence by producing a coloured optical glass.

Calibration Constants

Lastly it must be remembered that any subsequent treatment of the photograph for survey purposes depends primarily on an explicit or implicit knowledge of the relative positions of the plate and the internal perspective centre. This reduces generally to the determination of the length and the position of the foot of the perpendicular from the rear node on to the plate, or, alternatively, of the space co-ordinates of the rear node relative to arbitrarily chosen plate axes. It is absolutely essential that these "calibration constants" of the camera, once determined, shall remain constant under all conditions of exposure and to the same degree of precision as is inherent in the photograph. In cameras designed primarily for precision this implies rigid construction in metal of known coefficient of expansion, and the interposition of no detachable parts between the lens and the mechanical focal plane. Any system of indicating the plate axes to which these calibration constants are referred must be similarly of rigid construction and must

not vary between different photographs once the camera has been calibrated. Particular attention must also be paid to the mounting of the lens barrel in the camera body, and to the relative mounting of the lens components in the lens barrel. The latter should preferably be of reasonably thick steel, or some other metal possessing strength and having a coefficient of expansion of the same order as that of the lens components.

Summary of Requirements

In conclusion, I will summarize the precision requirements of those parts of the air survey camera which are of peculiar interest to the optical manufacturer, adding where necessary certain inseparable practical considerations.

The lens may be of focal length between 6 and 15 inches, although greater focal lengths than 10 inches are not likely to be required in the immediate future. To permit of a reasonably short exposure from the rapidly moving aeroplane it should work at a large aperture such as $F/4{\cdot}5$. In order that entirely similar lenses may be used in similar positions relative to the negative, in the camera and in the plotting machine, the various lenses in a batch should be within narrow limits of a mean focal length—the narrower the limits the better. The lens should be corrected in order of importance for coma; curvature, astigmatism, and central spherical aberration; and for distortion. To this end it should be designed for, and supplied with, a filter of a certain specified colour. If desired, the lens may be designed primarily for the outer field, and definition in the centre improved by figuring. Any money spent on such processes of refinement is money well spent, and is in any case only a very small fraction of the running costs of survey operations. Lastly the angular covering power should be as large as possible in order to economize in flying time and ground control, and to permit economical photography being carried out under clouds. In this connexion it may be remarked that a large aperture wide-angled lens of say forty-five degrees semi-angle would possess a considerable value in certain classes of survey, provided it could be designed within certain limits of aberration. It is not to be expected that a wide-angled lens of this nature would give the precision required for a contoured survey under average conditions at a medium scale, or for photographic extension of control. These are not the only applications of the method, however. It will shortly be possible to state definite tolerances for various classes of work, when it is to be hoped that attempts will be made to produce such lenses. Their commercial value is not open to doubt.

The shutter should be of a type designed to expose the whole of the plate at once, or nearly so. It should of course be optically efficient in order that as short a total exposure as possible may be given. Its most important qualities, however, should be durability and reliability. Time or money will seldom permit of re-photography of an area to fill in gaps caused by shutter jams, even if a little extra precision is gained by running the risk of such jams.

These requirements are of course contradictory. I can only hope that in this short paper you have been given a rough idea of their relative importance, in order that a reasonable compromise may be effected in the future design of precise aerial photographic equipment.

DISCUSSION

Mr T. Smith: It will facilitate consideration of some of the problems which have been suggested to-night if we are informed at what height the aeroplane is likely to be flying when photographs are taken, and whether this height will be varied when the field of view includes ground of very different altitude.

Major Cochran Patrick: Col. Winterbotham has asked me to reply to this question. No definite answer can be given, as it is entirely a question of scale, which again is dependent on both height and focal length.

I believe I am right in saying that it is common practice in ground survey work to prepare the field sheets on double the scale of the final map, and it is the aim of the air surveyor to work up his sheets before the final stage to a similar scale. In the interests of economy, however, it is advisable to photograph on as small a scale as possible provided that we can be assured that no excessive errors are introduced during the process, optical or otherwise, of converting the image on the negative to the fair mapping sheets.

In order to do this we have two courses open—one to fly higher, and the other to use short focal length lenses. We have come to the opinion that 16,000 feet is the highest altitude at which it is practical to operate, as operation at a greater height involves very considerable additional expense in engine power, etc. and the cost of suitable aircraft becomes prohibitive. Besides this we have difficulties of cloud and such interference to contend with, so that often our working height will have to be less. We can take it, however, that the majority of work will be done between 10,000 and 16,000 feet.

The approximate height of the camera being established, we choose our focal length of lens to suit the scale of work required. As I have already mentioned we aim at actually photographing on a smaller scale than the scale of the working sheets, and under present conditions we find that we can enlarge the negative three times without introducing excessive errors. This amount of possible enlargement might be increased, and in this we should appreciate advice. Lieut. Hotine mentioned the difficulty of obtaining a panchromatic emulsion with a sufficiently small grain, and I would like to emphasize that this is particularly important, as naturally the grain is enlarged with the detail, and in any considerable enlargement tends to confuse the detail.

Besides increasing the height from which we take our photographs, we can also economize by increasing the angular covering power of the lens which we

can use without introducing undue distortion. Lieut. Hotine has mentioned a semi-angle of 45°, but I think that we would be satisfied at the moment if we could obtain lenses, the total covering angle of which amounted to just over 60°. This figure has various attractions, one of these being that $60° \times 3$ makes 180°, and with three lenses we can sweep the visible ground from horizon to horizon.

Comdr. T. Y. Baker: In determining the position of the camera by mathematical methods the solution must lead to an algebraical equation of the eighth degree and there will be sixteen positions, real or imaginary, that the apex of the pyramid can occupy relative to the surveyed ground triangle. Some of these positions are obviously irrelevant, but there may be cases in which it is hard to say which is the true solution and which must be rejected.

Mr H. C. Raxworthy: It would, I think, be interesting to know to what extent stopping down the diaphragm in projecting the aero-photogram has been used. I should imagine that, where the plate at the time of exposure has been tilted but slightly, good results might be attained, although what we have learned regarding loss of definition due to the grain of plates shows that microscopic sharpness is a great desideratum. The problem of retaining the angle of view of the original negative whilst the superposition is being made is very intriguing and one that lens computors ought to take up without delay.

Col. MacLeod: A small stop has been tried, but the consequent loss of light is too great.

Flight-Lieut. C. Porri: I would like to confirm Major Cochran Patrick's statement that Mr Smith's question is a difficult one to answer directly. So much depends on the scale required and the circumstances attending each individual survey. In a case of very high mountains approaching the "ceiling" of the aeroplane it was most probable that oblique photography would be employed, and in that case not only the actual height of the aeroplane had to be considered, but also its horizontal distance from the object to be photographed.

Capt. G. T. McCaw: In considering the calls which aerial survey makes on the optical expert, one may take first the lens. Since the scales of photographs are frequently greater than is required, the surveyor is demanding an orthoscopic lens of wider angle. For rectification he wants a lens which will function when the conjugate focal planes are considerably inclined to one another. Again as to the prism: the only simple prism in use is the camera lucida. It is time that this crude instrument were replaced by an apparatus which would not damage the human eye. When we come to higher systems, we must consider the replacement of the ordinary commercial stereoscope by an instrument taking in a wide field and adapted to the draughtsman's use; nor should he be expected to ignore headache. In still more complicated apparatus, of which some examples have appeared on the screen to-night, there is a wide field for the optical designer and manufacturing optician. It should surely be possible to replace these excessively complex

systems—this optical articulation with its almost unlimited openings for error—by something simpler, more direct, and more workable.

In attempting to answer Mr Smith's question, one must consider the conditions. Thus, for a cadastral map, the heights of features are wanted only in so far as they affect the planimetry. For topographical maps the question is to be asked, not so much in terms of absolute height of feature, as in terms of permissible error of height. Let us suppose that the maximum permissible error is one-fifth of the contour interval (*i* in feet), that the air base of a pair of vertical photographs is one-half of the height of the aeroplane, and that parallax cannot be measured within the limits of the circle of confusion, say, 1/200 inch; then, if the focal length be *f* in inches, the maximum height of flight is 20 *fi* feet. Since *i* is commonly the reciprocal (*r*) of the map scale divided by 1250, the maximum height of the aircraft can also be expressed as *fr*/60. Thus, if the lens be of 6 inches focal length and the map scale be 1 inch/1 mile, the maximum height should be about 6000 feet. If the photographs are obliques, the expressions above are to be multiplied by the cosine of the tilt. The rule of course is not absolute: it may be broken when circumstances dictate the necessity or as experience suggests a modification.

Mr W. M. Hampton: In connexion with Lieut. Hotine's remarks on the various aberrations to which his photographs are subject, one suggestion occurs to me, which, however, appears so obvious that I have some diffidence in putting it forward. If, as I expect, the method has already been tried, it will be interesting to know from Lieut. Hotine the reason for its rejection. He said that errors were introduced through the curvature of the plate by the tension of the emulsion. In this case, why not do what is done in the case of "non-curling" film and put an emulsion on both sides?

Lieut. Hotine: The curvature of the plate is found to be reduced only slightly by removing the emulsion. Further, it is not reasonable to suppose, in view of the comparative smallness of the curvature, that the pull of the emulsion had strained the plate beyond the elastic limit. I imagine, therefore, that the *main* trouble lies in the curvature of the glass apart from the emulsion. If a light, flat, and reasonably cheap plate could be produced in the first instance, I do not doubt that the tension of the emulsion could be equilibrated as Mr Hampton suggests.

Mr Hampton: If the trouble is really due to the curvature of the glass apart from the emulsion, why not use "patent plate"?

Mr Smith: It is to be hoped that the statement of the problems will lead makers of optical instruments to devise systems to meet at any rate some of the requirements. Although some of the problems are novel, it does not appear improbable that satisfactory solutions could be found.

Prof. A. F. C. Pollard (communicated): After listening to the able and lucid explanations of the requirements of photogrammetric apparatus, one feels that any suggestion of possible improvement will be welcome and perhaps be put to

the test. While discussing with Lieut. Hotine the possibilities of geometrically flat photographic plates, it occurred to me that such a flat sensitive surface might be prepared in one of two ways. If a thin celluloid film be slightly and uniformly stretched in all directions and its boundary be constrained to lie in a plane, it is evident that the whole film will be geometrically flat. Let, therefore, a frame be made like the rim of the lid of a cardboard box with suitable ribs arranged diagonally to strengthen the structure and standing well away from the plane in which the free edge of the rim lies. This edge can be lapped so as to lie in a plane.

The boundary of a thin photographic celluloid film can now be constrained to lie on this lapped edge, either by clamping it in the same way that a lady clamps her embroidery between two hoops, thereby slightly stretching it in the process, or the edge can be smeared with very dilute shellac varnish and pressed against the back of the film at a temperature higher or lower than that at which it has to be used. If the difference in coefficients of expansion of the celluloid film and the material of the frame is small, then upon attaining the working temperature the film will be slightly stretched. The low temperatures encountered in aerial photography, however, may render the latter method impractical, but nevertheless it might be useful to astronomers.

I am proposing of course that the film is never separated from its frame during exposure, development, and measurement, and is not unlike an engineer's surface plate in form. A difficulty arises however in the choice of material for the frame. Sheet celluloid has a mean coefficient of expansion of 109×10^{-6} between 20° and 70° C. and the only material which occurs to me at the moment to use for the frame for a first trial is ebonite, which has a mean coefficient of expansion of $84{\cdot}2 \times 10^{-6}$ between 25° and 35° C. Ordinary ebonite will not keep its shape for long, but it might be possible to find some composition which has the same coefficient of expansion as the celluloid film used and would not change its shape with time. This photographic "flat" must of course be free to expand or contract in its own plane when fixed in the camera during exposure.

I would very much like to know if such a method of obtaining a flat sensitive surface has ever been tried; if it has not, perhaps it would be worth while investigating the possibilities.

Lieut. Hotine: In reply to Major Cochran Patrick, I should perhaps explain that by a semi-angular field of 45° I mean the angle subtended by the semi-*diagonal* of the plate, this being the dimension which influences the design of the lens. A tri-lens camera to sweep the visible ground from horizon to horizon would require a semi-angular field of at least 30° in a plane perpendicular to the adjacent edges of the three plates. To allow for accidental tilts and a small overlap between the three photographs, this would not amount to much less than 45° in the direction of a diagonal, though the actual figure would of course depend on the shape of the plates. Such a camera would be of use as a means of determining the tilts of all three photographs from the images of the visible horizon—a process which is being successfully practised in Canada. In stating the desideratum of 45°,

however, I had in mind more the necessity for economy in single vertical-axis photographs, since the economy is largely lost in the case of wide-angle obliques where much of the detail is too distant for accurate plotting. In this connexion, the figure of 45° should be taken as the ultimate end rather than the immediate necessity.

Mr Hampton's helpful suggestion of double-coated "patent plate" will certainly receive full consideration. The only possible objection to it that I can see will be on the score of cost.

Prof. Pollard's suggestion should be tried. The irregular distortions of film are not explained satisfactorily by the properties of celluloid alone, and it remains to be seen whether these would be eliminated by initial tension. It would obviously be unwise to jump from the frying pan of curvature errors into the fire of film distortion.

Further, there is a mean contraction of the photographic image, during development and drying, of the order of 0·2 per cent. If this is due to the celluloid base and does not result in rupture of the film, a strain of this magnitude translated in terms of stress in the frame might necessitate bulk and weight which the exigencies of rapid automatic plate-changing in the air would not allow. If, on the other hand, this contraction is due to slip of the emulsion, it is reasonable to suppose that even greater errors of distortion would result. A practical test of the idea is a simple matter, however, and will establish the exact force of these possible objections.

THE INTERFEROMETER PATTERNS DUE TO THE PRIMARY ABERRATIONS

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(Beit Research Fellow, Applied Optics Department, Imperial College of Science and Technology.)

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ABSTRACT. In order to obtain a rapid estimate of the quality of a lens by observing its behaviour in the interferometer, it is necessary for the observer to be familiar with the characteristic patterns of the various primary aberrations under various conditions of adjustment. A method is described by which these patterns can be easily calculated for any required case; and series of typical patterns are shown together with actual interferograms for comparison.

Introduction

Those who make extensive use of the interferometer for testing lenses come to recognize certain types of patterns which frequently occur, sometimes singly and sometimes mixed, and a working knowledge of the patterns due to various primary aberrations will be of great assistance in providing a rapid visual estimate of the quality of a lens, as they enable the observer to interpret a pattern at once as being due to certain aberrations in certain proportions. It is the purpose of this paper to show from theoretical considerations how the pattern due to any given aberration or mixture of aberrations can be accurately plotted, and many cases of pure spherical aberration, coma, and astigmatism have been completely worked out and are included in the paper.

The Fundamental Equation

The method of determining these patterns is based on Professor Conrady's formula for Optical Path Difference (O.P.D.). We will consider parallel light falling on the front surface of a lens such as a small telescope objective, and arriving at a point M (Fig. 1) displaced from the ideal paraxial image-point P in the Petzval surface by an amount δl along the axis of the lens beyond the ideal image, δh in the direction of increased height h of the image, and δt perpendicular both to δl and δh. Professor Conrady has shown that, so far as primary aberrations are concerned, the difference between the optical path of the light reaching this point via O, the centre of the lens, and via a point Q in the lens aperture with

co-ordinates (x, y), the positive direction of y being the same as that of the image-height h, is given by the expression

(Path via centre) − (path via the point Q)

$$= \text{O.P.D.} = \frac{a_1}{4}(x^2+y^2)^2 + \left(\frac{h}{l}\right) a_2 y\,(x^2+y^2) + \frac{1}{2}\left(\frac{h}{l}\right)^2 a_3\,(x^2+3y^2) + \frac{1}{2}\frac{\delta l}{l^2}(x^2+y^2) + \frac{\delta h}{l}\,y + \frac{\delta t}{l}\,x \qquad \ldots\ldots(1).$$

In this formula l is the distance of the image from the back surface of the lens, or in a telescope objective nearly enough equal to the focal length f; a_1, a_2, and a_3 are functions of the radii, refractive indices, thicknesses, and separations of the lens and are therefore absolute constants for a given lens system of given construction and size. They are known as aberration constants, a_1 being a measure

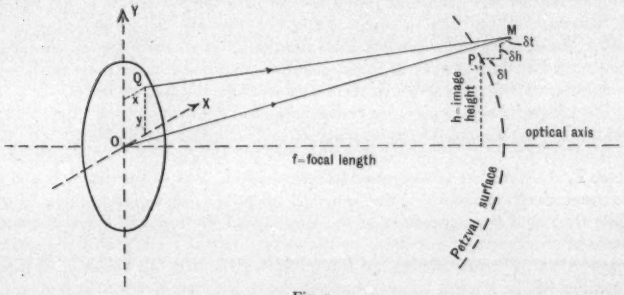

Fig. 1

of the spherical aberration, a_2 of coma, and a_3 of astigmatism; the values of these being connected with the usual geometric measures of the aberrations by the following approximate relations:

$$\left.\begin{aligned} a_1 &= \frac{\text{Longitudinal (primary) spherical aberration}}{s^2 f^2} \\ a_2 &= \frac{\text{Coma}}{s^2 h} \\ a_3 &= \frac{\text{Distance between focal lines}}{2h^2} = \frac{1}{2f} \end{aligned}\right\} \qquad \ldots\ldots(2),$$

"coma" being here understood to mean the lateral distance between the paraxial image-point and the intersection of the rays from the two extremities of the y-axis of the lens, and s being the semi-aperture. These equations apply exactly to a pencil of light passing at small inclination through a thin lens of small aperture, but can be taken as approximately correct even if the lens departs somewhat from these conditions.

The other two primary aberrations, curvature and distortion, do not affect the definition of an image but only its location, such aberrations coming into equation (1) in the terms in δl and δh respectively.

When a lens is placed in the lens-testing interferometer, the pattern seen is as it were a contour map of the wave leaving the lens aperture, on which each band passes through all the points having the same O.P.D. Moreover, since the light has passed through the lens twice, the O.P.D. of any one band is half a wavelength more or less than the O.P.D. of the next band on either side of it. Equation (1) therefore becomes the equation to the whole family of curves constituting the pattern seen in the interferometer, the various members of the family being obtained by assigning to the O.P.D. a series of values differing by $\lambda/2$.

Hence in order to predict the pattern which would be produced by a lens of given properties, when placed in the interferometer in correct adjustment, assuming also definite numerical values for the small displacements δl, δh, and δt, we substitute all our data in equation (1), and then plot the resulting family of curves, there being one curve for each additional $\lambda/2$ in the value of the O.P.D. Care must be taken to have all linear quantities expressed in the same unit, preferably the wave-length, although the values of x and y should be in cm.

We will write equation (1) for convenience as

$$P = A(x^2 + y^2)^2 + By(x^2 + y^2) + C(x^2 + 3y^2) + D(x^2 + y^2) + Ey + Fx \quad \ldots(3),$$

where P, A, B, etc. are all expressed in wave-lengths. The quantities A, B, and C are respectively measures of the spherical aberration, coma, and astigmatism; while D, E, and F are measures of the longitudinal, vertical, and lateral displacements of the centre of curvature of the convex mirror from the ideal paraxial image-point. When calculating the pattern for a given lens, the values of all these quantities will be known.

Methods of Plotting the Curves

The curves may of course be plotted in any manner the experimenter may desire, but there are two principal methods which will now be described. The Cartesian method is the most obvious, and will give every point on every curve with complete accuracy, but the polar method has been recently developed as it gives a complete family of even twenty or thirty curves in about the time taken to calculate one single curve by the older method. The polar method was used to determine all but the simplest of the patterns given in this paper, and is strongly to be recommended. The accuracy is not very high, but it is amply sufficient for every purpose. The Cartesian method has the advantage that by the usual methods of curve tracing, such as the determination of maxima, asymptotes, nodes, etc., the general trend of the family can be found without actually plotting it out; in fact it is very desirable to do this even if the polar method is to be employed finally.

(*a*) *The Cartesian method.*

It is clear that, for a given value of P, equation (3) expresses y as an implicit function of x, or vice versa. Hence if we assign a value to one variable, we have an equation for calculating the corresponding value of the other variable for any given value of P. The point obtained by plotting the values of the variables will lie on the curve joining all the points having that O.P.D. given by the chosen value of P. Another value of the first variable will then enable another point on this curve to be determined, and so on until sufficient points have been plotted to enable the curve to be drawn in free-hand. It can easily be seen how laborious this method will become if many curves are required.

(*b*) *The Polar method.*

This is essentially a graphical method of determining the values of one variable which correspond to assigned values of the other, but for convenience the equation is converted into polar co-ordinates. By putting $x = r\cos\theta$ and $y = r\sin\theta$ into equation (3) we get

$$P = Ar^4 + Br^3\sin\theta + Cr^2(1 + 2\sin^2\theta) + Dr^2 + Er\sin\theta + Fr\cos\theta \quad \ldots\ldots(4).$$

We now choose a series of values of θ, and generally 0°, 20°, 40°, etc. up to 160° will be found convenient. Then for each value of θ an equation is obtained of the type

$$P = Ar^4 + Gr^3 + Hr^2 + Jr \quad \ldots\ldots(5),$$

where A is a measure of the spherical aberration,

$G = B\sin\theta$, of coma,

$H = C + D + 2C\sin^2\theta$, of astigmatism and longitudinal displacement,

$J = E\sin\theta + F\cos\theta$, of lateral displacement.

By assigning a series of values for r, corresponding values of P may be calculated by equation (4), and a small auxiliary graph plotted connecting P and r for that particular value of θ, taking P as ordinates and r as abscissae. Values of r are read off from this curve for each of the ultimately required values of P, say at $P = \frac{1}{2}\lambda$, λ, $\frac{3}{2}\lambda$, etc., until r becomes equal to the semi-aperture of the lens. This process is repeated for each of the selected values of θ, and the final results are marked off along radial lines on the final drawing. The whole family of curves can then be immediately obtained by joining up the corresponding dots, or dots having the same value of P, on all the radial lines. To save the trouble of reading off the values of r at the chosen values of P, it is convenient to plot the auxiliary graph to the same scale for r as will finally be employed, and to place the r-axis of this curve over that particular radial line of the final drawing corresponding to the value of θ being used. The points required can then be directly pricked right through into the final paper beneath, without the necessity of taking any readings from the auxiliary graph at all.

With this method the amount of work is really very small, especially if care is taken to choose a suitable range of values of r for substitution in equation (4)

when building up the auxiliary graph. Moreover with this method the ultimate amount of work to be done depends hardly at all on the number of bands required in the final pattern, while by the direct Cartesian method every single band has to be plotted out point by point, and the result is quite unnecessarily accurate.

The Characteristic Patterns for Primary Aberrations

For the calculated patterns given in this paper, an imaginary lens is assumed of 25 cm. focal length and 3 cm. aperture, supposed to be viewed in the interferometer with the mercury green light ($\lambda = \cdot 00005461$ cm.), the direction of the incident beam being inclined at 5° to the axis of the lens. We will suppose also that this lens is such as to have

$$a_1 = \cdot 00008 \text{ cm.},$$
$$a_2 = \cdot 00125 \text{ cm.},$$
$$a_3 = \cdot 01 \text{ cm.}$$

These values correspond roughly to aberrations of the marginal ray at the best focus, of 2, 7, and 5 times the Rayleigh quarter-wave limit respectively. Of course the number of possible patterns with such a lens is unlimited, and most of them are quite uninteresting. To be able to interpret the interferences produced by a lens under test, the observer should be quite familiar with the characteristic patterns due to the various aberrations taken separately; these will now be given.

On substituting these assumed properties of our lens in equation (1), we get the computing equation for this lens as

$$P = \cdot 4\,(x^2 + y^2)^2 + 2y\,(x^2 + y^2) + \cdot 7\,(x^2 + 3y^2) + [14 \cdot 65\delta l\,(x^2 + y^2) + 732 \cdot 4\,(y\delta h + x\delta t)] \quad \ldots\ldots(6),$$

where P and all the coefficients are expressed in wave-lengths.

(*a*) *Pure spherical aberration.*

By equation (2) we see that, with the assumed value of $a_1 = \cdot 00008$, the longitudinal aberration of this lens, or the distance from the marginal to the paraxial focus, would be $\cdot 11$ cm. Hence patterns have been calculated for $\delta l = \cdot 1$, 0, $-\cdot 1$, and $-\cdot 2$ cm. (Fig. 2). The patterns in the upper row of that diagram were obtained by putting $\delta h = \cdot 002$ in each of the four cases, these families being plotted by the polar method. For example, in the case where $\delta l = 0$, $\delta t = 0$, $\delta h = \cdot 002$ we have as equation to the family

$$P = \cdot 4\,(x^2 + y^2)^2 + 1 \cdot 56y,$$

or in polar co-ordinates, $\quad P = \cdot 4r^4 + 1 \cdot 56r \sin\theta.$

Auxiliary graphs were constructed for each 20° in the value of θ, and the abscissae corresponding to $P = 0$, $\pm\frac{1}{2}$, ± 1, $\pm\frac{3}{2}$, etc. pricked through as described above. On joining up all the points having the same value of P, the required curves were obtained.

The photographs given below the drawings in Fig. 2 are actual interferograms

obtained with a lens having spherical aberration, to show how nearly the actual patterns resemble those calculated for pure primary spherical aberration. The departures from the theoretical curves are due to the lens used not having precisely the amount of aberration assumed in making the calculations.

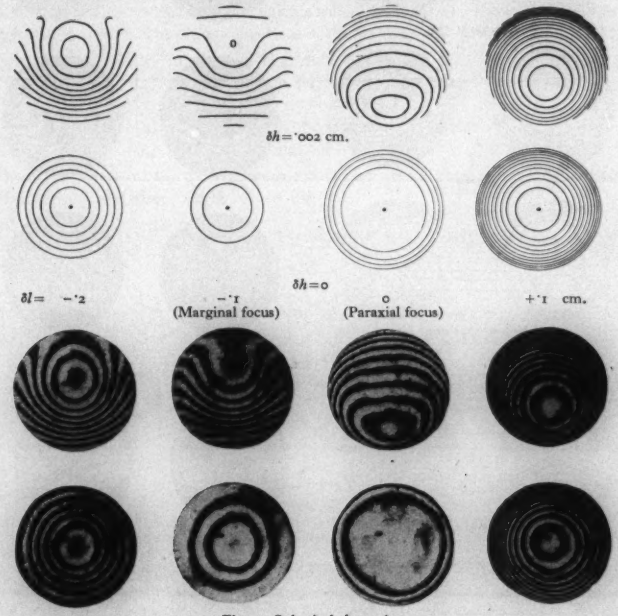

Fig. 2. Spherical aberration

(*b*) *Pure coma.*

For this case the equation to be plotted is

$$P = 2y\,(x^2 + y^2) + [14{\cdot}65\delta l\,(x^2 + y^2) + 732{\cdot}4\,(y\,\delta h + x\,\delta t)].$$

Twelve cases were here calculated, six with $\delta l = 0$, i.e. with the centre of the mirror coinciding with the paraxial image-point, and six with the centre moved away to a distance $\delta l = {\cdot}136$ cm. For each value of δl, the six patterns calculated were those given by $\delta h = 0$ or $\pm\,{\cdot}0041$ cm. and $\delta t = 0$ or ·002 cm. The results of calculation, together with interferograms for comparison obtained with a good telescope objective (focal length = 12 inches) tilted by a few degrees, are shown in Figs. 3 and 4.

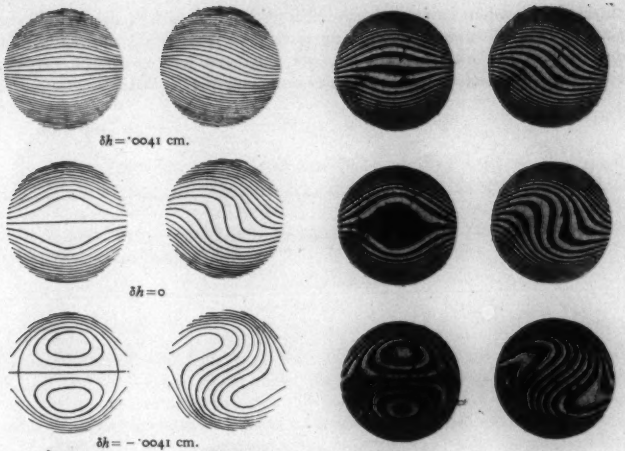

Fig. 3. Coma ($\delta l = 0$)

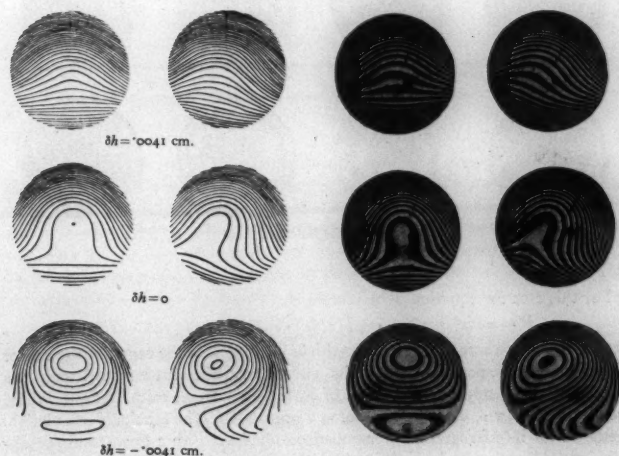

Fig. 4. Coma ($\delta l = \cdot 136$)

(*c*) *Pure astigmatism.*

Here we have the equation

$$P = \cdot 7\,(x^2 + 3y^2) + [14 \cdot 65\delta l\,(x^2 + y^2) + 732 \cdot 4\,(y\,\delta h + x\,\delta t)].$$

Patterns were calculated for five different positions of the mirror along the axis of the lens, with another pattern calculated in each case to show the effect of a slight tilt $\delta h = \cdot 002$. The chosen positions of the centre of curvature of the mirror are

(1) in the Petzval surface, $\delta l = 0$,
(2) coincident with the sagittal focal line,
(3) at the midway or best focus,
(4) coincident with the tangential focal line,
(5) within the tangential focal line.

These patterns, and the interferograms obtained with a good coma-free telescope objective slightly tilted, are shown in Fig. 5.

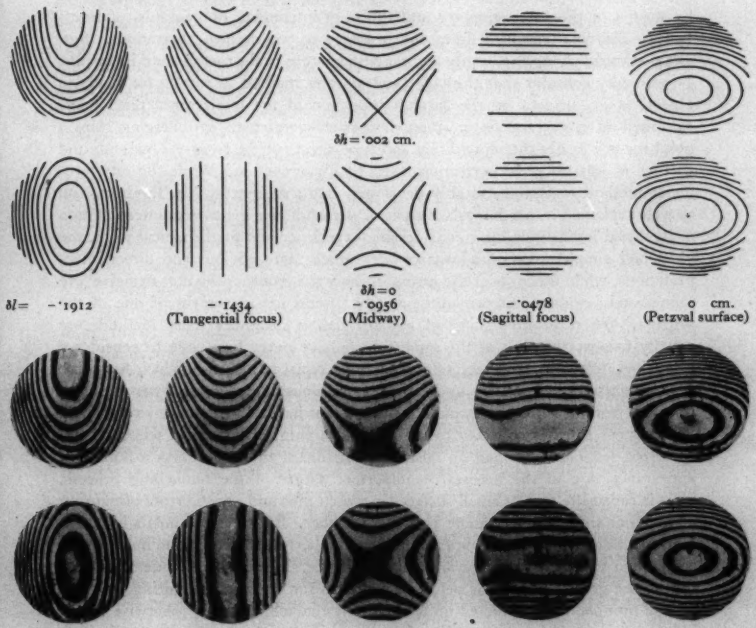

Fig. 5. Astigmatism

It is hoped that a knowledge of these patterns will greatly facilitate the testing of lenses with the interferometer, and all who perform such tests are urged to become quite familiar with them.

DISCUSSION

Prof. L. C. Martin: Mr Kingslake's results are an excellent example of the profitable nature of the actual numerical calculation of such formulae as those exemplified in the paper. The results are bound to prove very useful. At the same time it will remain difficult to obtain any very certain knowledge of the relative amounts of different types of aberration, which may be present together, without a careful analysis of the pattern; I think this matter has already received Mr Kingslake's attention.

Mr H. Dennis Taylor: I must congratulate Mr Kingslake upon the interesting nature of his exposition of the method of testing optical surfaces and objectives by means of the interferometer and the very interesting photographs of interference diagrams resulting from the presence of well-known aberrations. But, while acknowledging the beauty and scientific nature of the method and its results, I personally consider that the older and simpler method of testing for faults in objectives or surfaces by the careful inspection of the interference phenomena exhibited in cross-sections of cones of rays converging to or diverging from a point focus is really the more direct and time-saving of the two rival methods and ensures at least as great perfection being finally arrived at. While the interferometer method of testing, so ably developed into a system by Mr Twyman, and so well explained in Mr Kingslake's paper, demands, for the correct interpretation of its visual indications, very considerable experience and mathematical work, the older and simpler method admits of very much more obvious and direct interpretations, while the *signs* of the errors in the wave-front, positive or negative, are immediately evident independently of any to and fro movement of one of the mirrors. Wherever retouching of any surfaces is necessary, both methods are equally trustworthy, but at the same time I have never been able to consider a retouched reflecting or refracting surface quite equal to a surface which has come out perfect by first intention under the general polishing process. When the light reflected from or refracted through a surface that has been extensively retouched, however carefully, is collected to the focus of a point of light by a telescope and the cross-sections examined on either side of focus, there is always a flocculent appearance due to the necessarily imperfect nature of the retouching process. This is especially true of small surfaces in which plus and minus errors of figuring or material are necessarily more crowded together. But for instruments in which high magnifications are not expected the residual errors of retouching may be said scarcely to affect the definition to a serious degree. I have always understood that during the great demand for pentagonal prisms for rangefinders during the war, a large number were rendered quite usable by retouching which otherwise would have been rejected as quite useless.

Mr H. H. Emsley: The similarity between the curves plotted and the photographs is striking and I should like to ask Mr Kingslake (1) how the lenses used for the photographs were chosen or alternatively what is the procedure in choosing the coefficients in the equations so as to obtain this agreement between curves and photographs, and (2) what were the apertures and focal lengths of the lenses? Were they special in any way?

It will be very interesting for students to repeat the author's method in a few cases and so obtain an idea of the connexion between the contours and the aberration present.

Mr J. Guild: I think Mr Kingslake is to be congratulated on turning his attention to a subject which is becoming of increasing importance and will undoubtedly receive increasing attention from optical workers in the near future. In my view the interference method is destined ultimately to become the standard method of optical testing. In reference to Mr Dennis Taylor's comments, it is of course difficult at first to interpret the interference phenomena in terms of the geometrical aberration coefficients, but this difficulty becomes less as investigations such as Mr Kingslake's become available. The real utility of the interference method is that there is no possible doubt as to the exact phenomena which require to be interpreted. The fringes are there, irrespective of the observer's experience or education. Everyone sees the same thing and in such circumstances correct interpretations will sooner or later follow. Now by methods such as the star test there is not only the problem of interpreting the phenomena which the observer thinks he sees, but of being certain what is seen. There is no doubt at all that the detection of defects in the star disc is very largely a question of long experience; a skilled tester will see things at once which a less practised observer will overlook. On the other hand a fringe system is a picture in bold outline which gives the same information to everyone. The interpretation of this information is not likely to prove very difficult where the problem is faced.

I would like to compliment Mr Kingslake on his polar method of plotting these curves. Practical devices which save considerably the time involved in making any calculation are too often overlooked by persons engaged on optical work with the result that much valuable time and energy are wasted in routine and mechanical processes which might otherwise be spent on making progress with fresh problems.

Mr T. Smith: It is evident that the polar method of plotting described by Mr Kingslake is much more convenient than Cartesian or semi-Cartesian methods for this problem. It may be noted that the polar method is also preferable in analysing observed fringes to separate the various aberrations present. The method to be followed is to analyse an individual fringe by what in the limit becomes the Fourier method of analysis, and effect any further separation necessary by comparing results for a few separate fringes. Theoretically the method should enable aberrations of far more orders than have ever been discussed to be determined.

In two respects the nomenclature used by Mr Kingslake might be modified. The use of primary in the sense of first (or third) order aberrations should be

avoided since the entirely distinct division of aberrations into primary and secondary, corresponding to the older terms tangential and sagittal, is now widely recognised. Moreover it is better to speak of aberrational coefficients than of constants, as I remember being advised by a friendly astronomer many years ago.

Mr Kingslake: Prof. Martin is right in saying that a careful analysis of the pattern is necessary to determine accurately the amount of aberrations present, and I have attempted such an analysis on two or three occasions. There are, however, a vast number of practical difficulties in the way, making it an extremely laborious process to obtain an accurate analysis of the pattern even for primary aberrations, the amount of work involved increasing enormously if any attempt is made to include secondaries as well. The purpose of the present work was to enable an observer to say definitely that a given lens is, say, coma-free up to 4° from the axis, or that another lens has slight coma or astigmatism on the axis, showing decentering or irregular polish of the surfaces. With a little practice it becomes easy to say, after a few minutes' study of the behaviour of a lens in the interferometer, that the aberrations are so many times the Rayleigh limit, or that coma predominates over astigmatism, or that the coma is so slight as to be swamped by the spherical aberration, and so on.

Mr Dennis Taylor appears to have rather misunderstood the purpose of the present paper. The lens interferometer was originally developed by Mr Twyman to facilitate the retouching of telescope lenses, but I am convinced that the instrument has far more extensive uses than this, one of which is undoubtedly the measurement of oblique aberrations.

In answer to Mr Emsley's query, the lenses used were from Cooke's pathological set, and were of about 1½ ins. diameter and 12 ins. focal length. As regards spherical aberration, I found that lens No. 6 gave suitable patterns, then ascertained by a few trials what value of a_1 gave similar patterns, and proceeded to calculate the whole set with this figure. For coma and astigmatism, the patterns were calculated first, with values of a_2 and a_3 chosen to give a convenient number of bands, and then imitated as nearly as possible with lenses Nos. 1 and 2, the amount of aberration being varied in each case by altering the tilt of the lens. No. 1 is an ordinary telescope objective giving coma if slightly tilted, while No. 2 is a coma-free lens giving pure astigmatism. A special mounting was attached to the interferometer lens-carrier by which the lens could be tilted conveniently up to about 5°.

I agree with Mr Guild in all he has said, especially in regard to the difficulty of deducing accurate information from the star test. An interferogram is a permanent and definite record containing within it all there is to be known about the lens under the conditions of obliquity, etc. existing at the time of the experiment. The great problem is of course how to interpret and translate the pattern into familiar measures of the aberrations, and this is at present a very troublesome process. Perhaps the day may come when the most familiar mode of expressing the aberrations will be by their interferometer patterns!

With reference to Mr Smith's comment on nomenclature, it has long been the custom in the Imperial College Optics Department to call the Seidel aberrations "Primary" rather than to specify the order, because the actual mathematical order of the terms expressing the aberrations varies with the mode of representation. For instance, longitudinal spherical aberration is proportional to the square of the aperture, transverse or angular aberration depends on the cube, while in terms of differences of optical path it is proportional to the 4th power of the aperture. The secondary spherical aberration in these three modes of representation is respectively proportional to the 4th, 5th, and 6th powers of the aperture, so that confusion is necessarily introduced if we ever attempt to specify the order of any aberration.

A TRICHROMATIC COLORIMETER SUITABLE FOR STANDARDISATION WORK

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(Optics Department, The National Physical Laboratory.)

MS. received, 14*th January*, 1926. *Read and discussed*, 21*st January*, 1926.

ABSTRACT. The paper describes a trichromatic colorimeter designed and constructed in 1922. The instrument is suitable for standardisation work, as it may be employed to obtain the specification of any colour whatever in a form which may be converted if desired to any fundamental basis of specification. The three working primaries of the instrument are obtained by means of filters. The mixing of the primary colours is effected by a periscopic prism which rotates past three stationary sectors. Provision is made for adding any of the primaries to the colour under test, where this may be necessary in order to obtain a match. In this way colours of higher saturation than it is possible to match directly may be dealt with.

The instrument to be described was constructed early in 1922 for work in connexion with colour standardisation which is being carried out at this Laboratory. Publication has been delayed, as it was felt desirable to await the conclusion of certain parts of the investigation and the determination of certain fundamental constants with the instrument before describing it. Unfortunately, pressure of other work, combined with the inherent complexity of the subject, has protracted the main investigation to an unexpected extent. As the colorimeter has, however, been exhibited on various occasions, both at the Laboratory and elsewhere, and as it is unlikely that any serious modification in its design will be called for by subsequent work, a description of it at the present time may be of interest.

The principle underlying additive trichromatic colorimetry is that colours of any possible hue and, within certain limits, of any possible saturation may be produced by mixing three colours—known as the instrument primaries—in suitable proportions. As the potentialities and limitations of trichromatic colorimeters are very imperfectly understood among technical colour workers, it may be well to indicate here the general considerations applying to such instruments.

In Fig. 1 is shown the locus of the pure spectral colours on a trichromatic colour chart. For an explanation of such a chart and its uses the reader is referred to a previous paper which the author read before the Society*. Every possible colour sensation is represented by a point on this chart, the fundamental property of which is that all the colours producible by mixing two constituent colours in various proportions are represented by points on the straight line joining the points which

* "The Geometrical Solution of Colour Mixture Problems," *Trans. Opt. Soc.* 26 (1924–25) 139.

represent the constituents. A corollary from this, as shown in the previous paper, is that all the colours producible by the admixture of three constituent colours lie within the triangle formed by the lines joining the constituents two at a time. In general all the colours which may be produced by mixing any number of constituent colours in varying proportions are located on the chart within the smallest polygon (without re-entrant angles) which contains all the constituents.

Since all colour sensations are due to the admixture, in various proportions, of the spectrum colours, it follows that all possible colours are located in the area bounded by the curved spectrum locus and the straight line joining its two extremities. We shall term this area the colour field. Its precise shape and location on the chart depend of course on the particular set of trichromatic primaries for which the chart is constructed, but the essential characteristics of the field and the colorimetric relationships between different points in it are unaffected by such purely mathematical transformations.

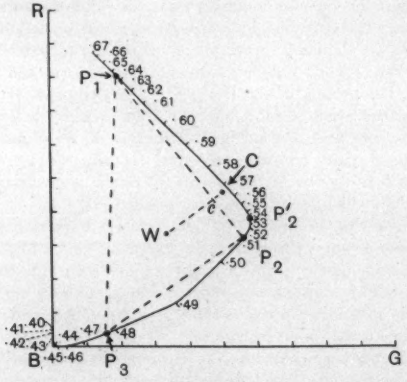

Fig. 1

It is at once evident from the shape of the colour field that it is not possible to find any three primary colours such that suitable combinations of them will match all other colours; for no triangle having its corners within the colour field can contain the whole field.

We thus see at the outset that no trichromatic colorimeter can be constructed which will match every possible colour. What then are the limitations of such an instrument?

Suppose we choose as our instrument primaries three fully saturated colours such as P_1, P_2, and P_3 (Fig. 1). With such primaries it is possible to match all colours within the triangle $P_1P_2P_3$, but colours within the three regions of the colour field which lie outside the triangle cannot be matched. For example, consider the colour

C: the most saturated colour of the same hue which can be produced with these primaries is c, which may be produced by mixing P_1 and P_2 alone. Any admixture of the third primary would increase the difference between the resulting colour and C. The sides of the triangle, which are the loci of mixtures of two only of the three primaries, locate the most saturated colours which can be matched with the instrument. This explains why spectrum colours cannot in general be satisfactorily matched by a three-colour mixture: in the regions between the primaries the best match obtainable is less saturated—paler—than the spectrum colours.

By suitable choice of primaries we can improve the saturation obtainable in one region at the expense of other regions. Thus if our green primary be taken at P_2' (wave-length ·53 to ·54 μ), one side of the triangle will lie very close to the spectrum locus and good matches in the yellow region of the spectrum will be obtained. On the other hand, conditions in the blue-green region will be made considerably worse. Conversely, by reducing the wave-length of P_2 we improve the saturation of the blue-green at the expense of getting very pale yellow and orange.

It is clear that, whatever points in the spectrum we select as P_1, P_2, and P_3, the same general inability to match spectrum colours, except for restricted regions of the spectrum, will be found. Further, so long as the primaries can be mixed so as to match white, i.e. so long as the triangle $P_1P_2P_3$ contains W, it is possible to produce colours of every possible hue; for a line from W to any point in the periphery of the colour field is a line of constant hue, with a saturation increasing from zero at W to its greatest possible value at the periphery. The point where any such line cuts a side of the triangle represents the colour of greatest saturation, for the hue in question, which the primaries can produce.

These two properties, viz. ability to produce colours of every possible hue, and inability to produce colours of full saturation except possibly in limited regions of the spectrum, being common to all sets of primaries which fulfil the condition of being miscible to match white, there is obviously no ground for the belief that any set of primaries is unique in possessing some property which renders it peculiarly suitable for use in a three-colour instrument.

Much difficulty has been caused to colour workers in dealing with trichromatic systems by the conclusion of the late Sir Wm. Abney* that special significance attaches to the position of the primaries, particularly the green primary. This he claimed should be located at the precise point in the spectrum where a "green-blind" person sees grey. According to his experiments it is possible, when this green primary is used, to match all spectrum colours provided the latter are reduced in saturation by the addition of white, but this cannot be done if a different green primary be selected.

This last statement is wrong, as is obvious from what has already been said. If to any colour, such as C, which lies outside the triangle $P_1P_2P_3$, we add a progressively increasing amount of white, the resultant colour will move along the line CW. When it has reached, or passed, c, it can be matched.

Similarly for any other set of primaries or any other colour, we can always by

* *Colour Vision* (1895) p. 92.

the addition of white produce a colour of the same hue but of diminished saturation which will lie inside the triangle and which can therefore be matched. Thus Abney's criterion for suitability of primaries applies to any set of primaries which can be combined to make white and is not confined, as he supposed, to a particular set. The choice of primaries has to be based on other considerations which we shall discuss later.

In practical colorimetry the measurement of a colour of higher saturation than can be directly matched may be accomplished by adding sufficient white to desaturate the colour until it comes within the working triangle. This, however, is merely a special case of the general method, which may be stated thus. Add to any colour which itself lies outside the working triangle any other colour which modifies it so that the resultant lies inside the triangle. Measure this modified colour in terms of the instrument primaries, then exclude the colour under test from the field of view, leaving only the added colour. Measure this in terms of the primaries and subtract the measurements from those obtained for the combination. The result gives the expression for the colour under test. Thus, let the readings of the in strument for the *modified* colour be

$$aP_1 + a'P_2 + a''P_3,$$

and let the readings for the *modifying* colour alone be

$$bP_1 + b'P_2 + b''P_3.$$

Then the readings corresponding to the colour under test will be

$$(a - b)\,P_1 + (a' - b')\,P_2 + (a'' - b'')\,P_3.$$

Since, by hypothesis, the colour is one that cannot be matched directly, one of these coefficients will have a negative value.

As special cases of the above procedure the modifying colour may be white, as already stated, or it may consist of one of the primaries alone, namely that which requires the negative coefficient in the final equation.

Thus, in the case of a colour such as C (Fig. 1) we could add to C sufficient of the primary P_3 to bring the resultant colour inside the triangle. The practical advantage of this procedure is that the subsequent evaluation of the modifying colour requires a setting of one primary only instead of all three as in the general case.

The Instrument

From the foregoing introduction it is evident that in a trichromatic colorimeter intended for general work it is necessary that provision be made for

(*a*) the illumination of the comparison side of the field of view with a variable mixture of three "primary" colours,

(*b*) the addition to the test side of the field of view of a modifying colour to desaturate the colour under test when this is of greater saturation than can be matched directly.

The methods by which these are accomplished in the present instrument will

be readily understood from the diagrammatic representation in Fig. 2, which shows a plan, and Fig. 2 *a* which is a view of that end of the enclosed portion of the instrument which is presented towards the light source.

Referring first to Fig. 2 *a*, the shaded areas represent sector-shaped apertures, each of about 59° angular extent, in the end of a box. These apertures are backed respectively by red, green, and blue gelatine filters mounted between glass plates.

The source of light consists of a 105-volt 100-watt "Fullolite" lamp*. This lamp has an opal spherical bulb of about 8 cm. diameter and constitutes an extended source of fairly uniform brightness over its whole area. The lamp is situated with its apex approximately at the focus of the outer zone of a condensing lens of 15 cm. aperture and about 30 cm. focal length. This throws a circular beam of light, which comes in effect from a broad source at infinity, on the end of the box

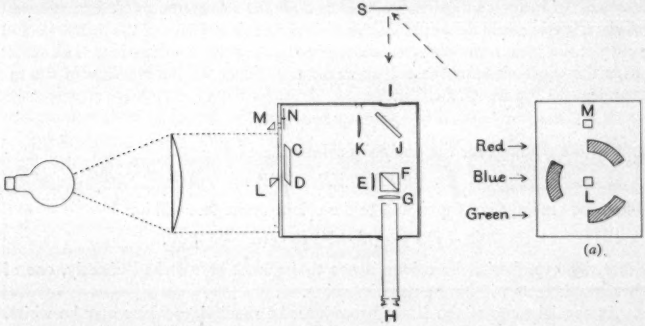

Fig. 2

which contains the coloured apertures. Inside the box a prism *CD* of the shape shown is mounted in such a way that it is capable of rotating about an axis *DE*. During rotation the end *C* passes each of the sectorial openings in turn. When *C* is opposite an opening, light enters the prism and, after internal reflection at each of its inclined faces, emerges along *DE*.

A lens *E* is placed so that the effective stop, a circular hole in the mount of the prism *CD*, is at its focus. *F* is a photometric prism to provide a matching field. *G* is a lens with its focus at *H*. An observer with his eye at *H* sees the reflecting section of the field of view illuminated by light from the particular sector which the end *C* of the rotating prism is opposite. Thus the colours red, green, and blue successively enter the eye. If the speed is sufficiently rapid the sensations mingle and a mixed colour is observed.

Each sectorial opening is fitted with a shutter, not shown in the diagram, by which the relative durations of the red, green, and blue stimuli may be varied, thus altering the proportions of these colours in the mixture.

* Made by Messrs Edison Swan Electric Co.

The colour to be matched enters by the aperture *I*. Light from this aperture fills the transmitting portion of the divided field. A lens of about 20 cm. focal length is fitted at *I*. If objects possessing structure, such as fabrics, etc., are placed near the focus of this, the structure is imaged at *H* and a perfectly uniform illumination is obtained in the field of view. Without the lens the structure is visible in the field of view and accurate matching becomes impossible.

In order that the colour under test may, if necessary, be desaturated sufficiently for a match to be effected, a plane parallel glass plate *J* is inserted in the test beam. Light from the source is reflected by the 45° prisms *L* and *M* through a hole in the end of the box. It passes through a regulating filter consisting of a circular lamp-black-in-gelatine wedge and then via the lens *K* and the reflecting plate *J* to the photometric prism and the observer's eye. The function of the lens *K* is to collimate

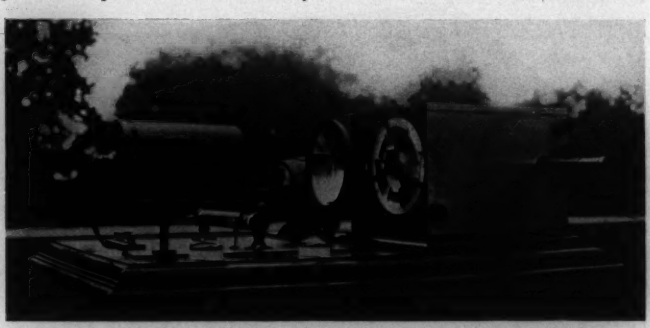

Fig. 3

the light from the aperture at *M*, so that an image of this aperture is formed at *H*. The light thus added gives a uniform illumination over the same portion of the field of view as is occupied by the colour under test.

By insertion of colour filters between the prism *M* and the hole the added light may be made of any desired colour. In particular it may be given the colour of any one of the three primaries by inserting a piece of the same filter as is used for that primary.

We may thus very conveniently add to the colour under test the unfiltered light from the source, any one of the primaries, or any other colour producible by means of filters.

A general view of the instrument is shown in Fig. 3*. The "Fullolite" lamp is housed in a lantern consisting simply of a horizontal tube open at one end. The

* The photograph was taken on the roof of the Laboratory in order to get suitable illumination. Needless to say, the instrument is not normally used in such a situation. All the parts are mounted on a large baseboard measuring 100 by 65 cm. approx.

lamp socket is fixed in the centre of the closed end, The condensing lens and other external features to which references have already been made are readily recognised in the photograph, which shows also the motor employed to drive the rotating prism, and a rheostat for regulating the speed of rotation.

The recesses at each end of the sectorial slots were provided to facilitate milling the essential portions of the radial ends perfectly straight. It is only, of course, a central strip of the slots (about 1 cm. in width) which is effective in supplying light through the rotating prism. The form of the boundary of the slots is therefore immaterial, provided the exposed ends are truly radial. These ends are bevelled to a knife-edge.

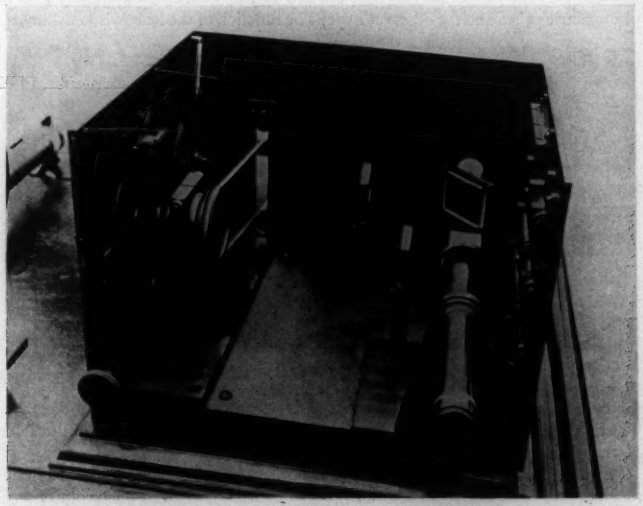

Fig. 4

The three shutters rotate about a central pin, stops being provided to limit their movements so that it is not possible, on the one hand, to proceed in the direction of opening one sector so far that the back end of the shutter encroaches on the next sector, or, on the other hand, to move the shutter sufficiently beyond the closed position that part of the sector becomes exposed at the back end of the shutter. The actual openings are cut to an angle of approximately 59° and the shutters to an angle of 61°. The leading edge of each shutter, which forms one boundary of the effective opening, is milled to an accurately radial knife-edge. Index marks are

attached to each shutter and read directly on scales engraved on the end of the box. The scales are engraved in units of half a degree, for convenience in dividing, but are numbered at 0, 5, 10, 15, 20, etc. The method of direct reading, despite the inconvenience of observing scales on the end of the apparatus, is regarded as much more satisfactory than the use of graduated drums which depend for accuracy on the behaviour of intermediate mechanism between the drums and the shutters. A further advantage of direct-reading scales is that the actuating mechanism, being relieved of the function of recording the position of the shutters, may be of the very simplest character. The arrangement adopted in the present instrument depends on gravity for moving the shutters in one direction, movement in the other direction being effected by stout linen threads which are wound on drums on the shafts of the operating handles seen on the right of the observing tube. Suitable counter-weights have to be fitted to two of the shutters so that gravity is effective at all sector openings. These weights will be easily recognised in Fig. 3. The arrangement of the threads will be clear from Fig. 3 in conjunction with Figs. 4 and 5, which show a side and top view of the interior of the box. The short lengths of square-threaded screw which are cut on the operating shafts carry blocks which rub loosely against the end of the box. A pin screwed into each block engages in the groove of the screw so that the block is carried along as the shaft rotates. Two stop pins are inserted in the screw near either end to limit the range of motion. When the pin in the block meets either of the stop pins, further rotation of the shaft in that direction is prevented. The pins are fixed so that the range of movement of each operating handle is just a little more than is necessary to move the shutter between its two limiting stops. This prevents inadvertent winding up of the thread until it breaks, or over-loosening until the thread begins to wind up in the reverse direction.

About three revolutions of the shafts are required to cover the range of shutter movement. This is found to give a convenient compromise between rapidity and sensitivity of control.

The other features shown in the diagram (Fig. 2) and referred to in connexion therewith will easily be recognised in Figs. 4 and 5. It will only be necessary to make further reference to one or two of these.

The vital part of the whole apparatus, as far as durability and reliability are concerned, is the rotating prism *CD* (Fig. 2). This prism is mounted securely in a metal housing attached to a plate which carries, at its opposite end, a balance weight consisting of a brass block. In each side of this block, and in its outer end, are fixed heavy-headed screws with lock nuts. One of these screws is seen in the photographs. The system must be carefully balanced, by adjustment of these screws, otherwise serious vibration will occur when the apparatus is in operation. Ball-bearings are employed. Needless to say, apertures are provided in the prism housing for the light to enter and leave, and the bearing shaft is tubular. After various trials of different materials, the best driving belt was found to be a thick cord carefully spliced. Two light vertical rollers are fixed at either side of the belt slot in the back plate. These are best seen in Fig. 5. They are to prevent the belt

striking against the sharp edges of the plate in the event of a lateral vibration starting up. The motor employed is of 1/30 H.P. and marked speed of 2400 ∾. The pulley on the motor shaft is about one and a half times as large as that on the peri-

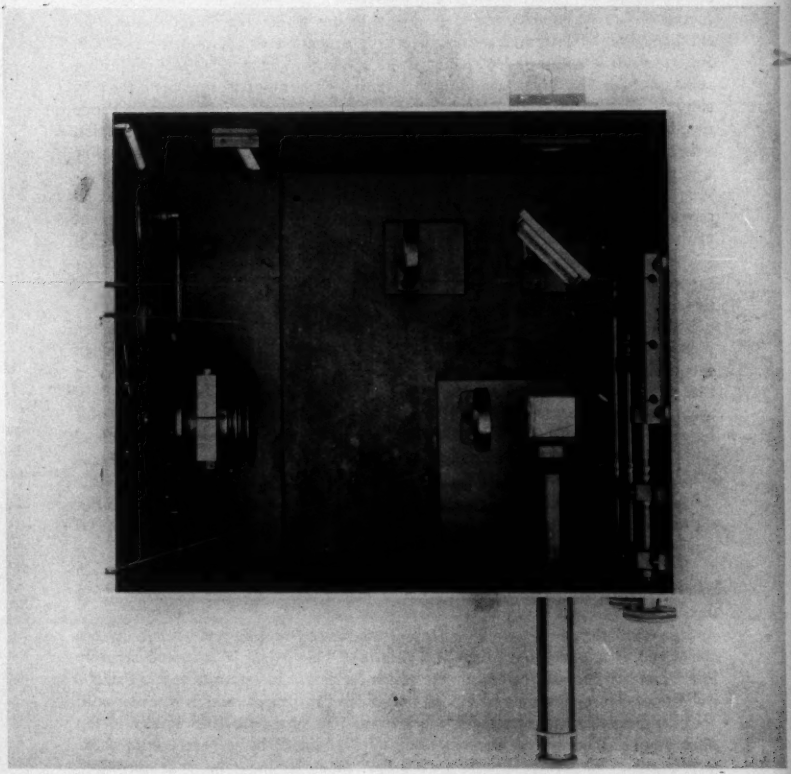

Fig. 5

scope shaft. A variable resistance, shown in Fig. 3, is provided to adjust the speed to a satisfactory value. In the left-hand back corner (Figs. 4 and 5) the lampblack annulus for regulating the amount of colour added to the test colour will be seen.

The density difference between the darkest and lightest settings of this annulus is about 2, giving a variation of 100 to 1 in the intensity of the light transmitted. When it is desired that no light whatever is supplied in this way, an opaque shutter is placed over the aperture. The annulus is not calibrated, and is used merely as a regulator, the evaluation of the added light being performed as subsequently explained. The annulus is operated by a thread passing round a drum on its axis and round another drum on the operating shaft seen at the bottom of the instrument. This shaft is similar in every respect to those used to operate the sector shutters. The milled heads of all the shafts are removable to simplify the fitting of the front plate. The sighting tube is telescopic. It is shown at its usual operating extension in Fig. 3, but is closed up in the other photographs. An iris diaphragm is fitted at the eye end of the tube. Suitable diaphragms are obtainable commercially from microscope manufacturers; they are intended for fitting above microscope objectives. The adjustable diaphragm, while not strictly necessary, is of considerable convenience, as explained later. When the sighting tube is at its normal extension, the diaphragm should be in the focal plane of the lens *G* (Fig. 2).

The photometric cube is built of two 45° prisms with one-half of their hypotenuse surfaces in optical contact. The line of division of the two portions is horizontal. A stop with a square opening of 9 mm. side is fitted between the cube and the lens *G* (Fig. 2). The field of view, therefore, consists of two horizontal strips 9 mm. long by 4·5 mm. wide. If the instrument is suitably adjusted the dividing line is practically invisible when the fields are matched.

As the eye is distant about 25 cm. from the field of view, the angular dimensions of the latter is about 2° square. A field of this size is chosen for two reasons: it lies almost entirely within the average "yellow spot" on the retina, and is of similar dimension to the field for which brightness measurements by the flicker method agree closely with other standard methods. It is of the greatest importance that quantitative work on the properties of the eye should all apply to the same region of the retina, and colour-matching apparatus with fields extending beyond the macula latea may give rise to results which are not at all representative of foveal vision. It was a favourite lecture experiment of the late Sir Wm. Abney to demonstrate this.

A simple two-part field, instead of a triple field, is used because the advantages of the latter are only realised when it is possible to have absolute equality of the illumination over all parts of the field. This is easily enough accomplished when the illumination of the duplicated field is due to a diffusing surface of appreciable extent, as is the case when measuring colours of papers, fabrics, etc., but an instrument of this kind may have to be used to measure the colours of the spectrum (in effect a coloured slit) or colours from the narrow capillary of a vacuum tube, etc. In general, under such conditions it is very difficult to arrange the auxiliary apparatus to give absolute uniformity of illumination over a large enough area to make the triple field effective. In such cases it is a hindrance rather than a help.

Choice of Primaries

The statements made in the introduction about the potentialities and limitations of trichromatic primaries make it evident that any set of three colours which form a triangle containing the white point on the colour chart may be employed as the working primaries of a colorimeter. The general procedure to be followed in using the instrument will be the same whatever set is adopted. It is not even necessary to employ fully saturated colours for the primaries, nor is it necessary to restrict ourselves to the familiar red, green, and blue combination. It is obvious from the colour chart that a colorimeter with purple, yellow, and blue-green as its primaries would involve no difference in principle and would require no modifications in the design of the instrument.

Our choice of primaries must be guided by practical considerations. These may vary where instruments for special purposes are concerned, but for general work we may lay it down that it is desirable to reduce as much as possible the regions of the colour field which lie outside the working triangle. This makes it practically desirable to use saturated colours as primaries. The same consideration dictates that the primaries should be near the corners of the approximately triangular spectrum locus. It is easy to see from the colour chart that primaries in the purple, yellow, and blue-green regions would be very bad indeed from the point of view of the amount of modification which most saturated colours would require in order to bring the resultant inside the working triangle. Clearly, the best primaries from this point of view are a red at, or near, the red end of the spectrum, a blue of wavelength in the neighbourhood of $\cdot 44\mu$ and a green somewhere in the neighbourhood of the sharp bend which occurs in the spectrum locus about $\cdot 53\mu$. It is not easy to locate *precisely* the optimum position of the green primary. The best position would be that which reduces to as small a value as possible the greatest error which may arise in the measurement of any saturated colour due to its lying outside the working triangle and having, in consequence, to be measured by a colour match made on a different colour for which liminal values of the various primaries may differ from those which apply to the colour itself. The writer has investigated the considerations which determine the best green primary for colorimetric work. These are of a quite definite character, but unfortunately some of the essential data required before a definite result can be stated do not appear to have been determined. It is not possible at the present time to say more than that the *RG* side of the working triangle should lie considerably nearer the spectrum locus than does the *GB* side, in order that the disadvantages arising from the necessity of modifying spectrum colours may be shared approximately equally by the two halves of the spectrum. It is probable, on such rough assumptions as have at present to be made, that the best position for the green primary is between $\cdot 52\mu$ and $\cdot 54\mu$.

In attempting to realise these primaries by means of filters further practical limitations are imposed on our choice. The complete set of the Wratten filters made by Messrs Kodak and the spectrum series made by Messrs Ilford were available for selection. There are several excellent reds. Of these Wratten No. 71

was selected. It gives a colour indistinguishable from monochromatic red of wave-length ·63μ. For the blue primary Wratten No. 49 B is used. This gives a fully saturated colour of wave-length ·45μ. The only green filter giving a fully saturated colour in the required region is the mercury green filter, No. 62 of the Wratten series, which has a hue (with the "Fullolite" lamp) of about ·537μ. This is almost certainly on the yellow side of the optimum, but is not inconveniently so. A filter having a similar transmission peak at a slightly shorter wave-length might be better, but there are none available at present.

Use of the Instrument

The use of the instrument is simple. Suppose the colour to be tested is that of an opaque object, such as a piece of coloured paper, cloth, etc. The specimen is placed at *S* (Fig. 2) and illuminated by whatever illuminant it is desired to use with it, the light being incident at approximately 45° to the normal*. The colour is then seen uniformly illuminating the test field. If the specimen possesses a smooth structureless surface, its distance from the lens *I* is immaterial, but if it has surface markings, it is desirable to put it at the focus of the lens so that these do not appear in the field of view.

If the colour to be tested is that of a transparent filter, a white screen, consisting of a plate which has been heavily coated with magnesium oxide by smoking it over burning magnesium ribbon, is placed at *S*, and the filter is placed up against the window *I*. An adjustable bracket, part of which is faintly visible in Fig. 5, is fitted to hold filters, or cells of liquid, etc. in front of the window.

Except for special purposes the illuminant which is supposed to be used for colorimetry is white light. While there is no universally agreed standard of white light it is the general custom among modern writers on colour to adopt as "white" a light having an energy distribution, within the visible spectrum, of a "black body" at a temperature of 5000° K. This has the same colour as average noon daylight at Washington, and is, roughly, of similar energy distribution. Daylight itself is, unfortunately, too variable in colour to be used for work of any importance, and it is necessary to use some source of greater constancy. All laboratory sources are of much lower colour temperature than 5000° K. and a bluish filter has to be used with them to obtain white light†. Having provided proper illumination for the test colour an attempt is made to match it by opening up the various sectors. If it is found possible to make an accurate match, the readings of the three scales are noted. These readings are converted to a standard basis as subsequently explained.

If it is not found possible to match the test colour, the nearest match will be one in which two only of the primaries are used. This indicates that it will be necessary to modify the test colour, as explained earlier. The filter of the same colour as the third primary is placed in front of the aperture at *M* (Fig. 2). The regulating

* See Appendix II (p. 124).
† See Appendix I (p. 122).

annulus *N* is adjusted until sufficient of the primary is added to the test colour to enable the fields to be matched. The sector readings are noted. A shutter is then placed over the window of the colorimeter to exclude the test colour, leaving only the added colour in the test part of the field. This is then matched in brightness by the primary of the same colour, and the reading noted. This reading is then subtracted from the reading of the same primary when making the colour match.

For instance, suppose the readings in a particular case were:

	Red scale (divisions)	Green scale (divisions)	Blue scale (divisions)
Match of test colour plus some of red primary	3	29	41
Match of added red primary alone	14	0	0
Readings corresponding to test colour alone	−11	29	41

In practice the observer should aim at adding just enough of the third primary to make a match possible. Had this been done in the above case the first red reading would have been 0 and the second 11. It is not always easy, however, to avoid adding just a little in excess of the minimum required, but the excess should be kept as small as is conveniently possible. As already stated we are not obliged to add the third primary as the modifying colour. We may simply add the unfiltered light from the "Fullolite" lamp. In this case the evaluation of the modifying colour will involve all three sectors, the readings being subtracted from the readings of the same sectors for the preceding colour match. Precisely the same result should be arrived at by either method.

If it is found that the test field is brighter than can be matched by the instrument, its brightness can be reduced either by increasing the distance of the illuminant from *S* (Fig. 2) or by inserting a rotating sector disc in the path of the beam. *No so-called neutral filter can be used for this purpose.*

If, on the other hand, the colour under test when illuminated as strongly as possible is of very feeble luminosity, the sector openings required to match it will all be very small, and the percentage accuracy with which they can be read will be correspondingly low. In such cases a neutral glass window which transmits about one-tenth of the light is placed in front of the lantern containing the "Fullolite" lamp. This, by reducing the light incident on the sectors to a tenth of its normal value, requires the openings to be ten times as great. We do not here depend on the neutrality of the glass, provided it is approximately neutral so as not to alter the *quality* of the primaries. We do not assume that all three are cut down to the same extent but determine separate constants for the scales when the neutral glass is in use. When the field brightness is high it is convenient to restrict somewhat the ocular pupil by means of the iris diaphragm. If the field brightness is low, as when measuring colours of low luminosity, such as navy blues, the darker shades of brown, etc., it is desirable to open the ocular pupil to the full diameter of the pupil of the eye so that the colour is seen under conditions as regards brightness similar to those which prevail in the ordinary use of the eye.

The scale readings of the various sectors are purely arbitrary. The usual convention in expressing results in terms of any trichromatic system is to choose the

units of the various primaries so that an equal number of units of each will make white when combined. To obtain our units we must therefore make a match of the standard white. The actual intensity of the white is immaterial since we are concerned with relative values only. From a set of readings for white we obtain factors for each scale to convert readings into trichromatic units.

In many cases it is the *quality* only of the colour with which we are concerned and the final results are expressed in a unit equation such as

$$C = aR + a'G + a''B,$$

where $$a + a' + a'' = 1.$$

In other cases, e.g. filters and coloured surfaces, we require information of a quantitative character, so as to distinguish between, say, a yellow and a brown, which might be identical in quality, differing only in the amount of light reflected or transmitted.

In such a case we first match the standard white, arranging the illumination of the screen so that the field brightness is as high as can conveniently be dealt with on the instrument. The filter—if it is a transparent colour that is in question—is then placed over the window of the instrument and the necessary measurements made, *the illumination on the screen being maintained at the same value as when matching the white.*

Let r_w, g_w, and b_w be the scale readings of R, G, and B sectors when matching white, and r_c, g_c, and b_c the readings corresponding to the colour. It is clear that r_c/r_w, g_c/g_w, and b_c/b_w measure the ratios of the amounts of each primary in the colour to the amounts of the same primaries in the incident white light. If, as usual, we specify the incident white light by a unit equation

$$W = \cdot\dot{3}R + \cdot\dot{3}G + \cdot\dot{3}B,$$

the transmitted light is represented both quantitatively and qualitatively by

$$\cdot\dot{3}\frac{r_c}{r_w}R + \cdot\dot{3}\frac{g_c}{g_w}G + \cdot\dot{3}\frac{b_c}{b_w}B.$$

Such an expression, in which the coefficients of the primaries do not add up to unity, is the *total equation* of the coloured light, taking the incident white light as unity. The sum of the coefficients measures the "quantity" of the coloured light. The complete equation is

$$\cdot\dot{3}\left\{\frac{r_c}{r_w}+\frac{g_c}{g_w}+\frac{b_c}{b_w}\right\}C = \cdot\dot{3}\frac{r_c}{r_w}R + \cdot\dot{3}\frac{g_c}{g_w}G + \cdot\dot{3}\frac{b_c}{b_w}B.$$

In testing opaque surfaces, the procedure consists of matching the standard white as before and then replacing the magnesium oxide screen by the coloured surface to be tested. It will be obvious that in this case the equations obtained do not give the ratio of the quantity of the colour to that of the incident light but to the quantity reflected by the magnesium oxide surface. Correction could be made for the reflectivity of the magnesia so as to compare the quantity of the colour with the quantity of the light which would be reflected by a perfectly reflecting and perfectly diffusing substance. This would be a more absolute basis for specification,

but unfortunately is not at present a practically satisfactory basis, as the reflecting properties of magnesia and other white substances are not very definitely known.

It is a better procedure at the present time to express quantities of reflected colours as fractions of the quantity reflected by a magnesium oxide surface used under definite conditions.

For some purposes it is desirable to interpret colorimetric results in terms of luminosities instead of trichromatic quantity units. To do this it is only necessary to know the relative luminosities of unit quantities of the primaries of the instrument.

To determine these constants we can proceed as follows. Three filters similar to those employed for the primaries of the colorimeter are carefully spectrophotometered, and the relative luminosities (for a normal eye) of the light transmitted by them from a source of definitely known energy distribution, such for instance as an Eastman Kodak standard acetylene flame or a standard electric lamp operated at a known colour temperature, are calculated by multiplying the ordinates of the spectrophotometer curves by the corresponding ordinates of the luminosity curve of the source, and measuring the relative areas of the reduced luminosity curves so obtained.

If the magnesia screen be illuminated by light of the standard quality for which these computations have been made, and one or other of the filters be placed over the window of the colorimeter, we obtain three colours of known relative luminosity, each of which may be matched by one only of the primaries. Let l_r, l_g, and l_b be the relative luminosities of these colours and let s_r, s_g, and s_b be the scale readings of the R, G, and B sectors when each separately matches the filter of its own colour, then l_r/s_r, l_g/s_g, and l_b/s_b measure the relative luminosities of a scale division of each primary. From this we can calculate the relative luminosities of a trichromatic unit of each primary, knowing the relative number of scale divisions in each of such units. It is usual to reduce these luminosity factors to such values that their sum is unity.

If L_r, L_g, and L_b are the luminosity factors determined as above for the three primaries, then a quantity of any colour represented by

$$\alpha R + \alpha' G + \alpha'' B$$

in trichromatic units will have a luminosity proportional to

$$\alpha L_r + \alpha' L_g + \alpha'' L_b.$$

Suppose, for example, we have a coloured filter for which the total equation is

$$\cdot 61 C = \cdot 18 R + \cdot 23 G + \cdot 20 B.$$

We can find from this the relative luminosity of the transmitted light as compared with the incident white light.

The luminosity of unit quantity of the white light is

$$\cdot\dot{3} L_r + \cdot\dot{3} L_g + \cdot\dot{3} L_b$$

or simply $\cdot\dot{3}$, if $L_r + L_g + L_b = 1$.

The luminosity of the transmitted light is

$$\cdot 18L_r + \cdot 23L_g + \cdot 20L_b.$$

The ratio of this to $\cdot\dot{3}$ is the relative luminosity of the transmitted light to that of the incident white light.

The same considerations apply to the colours of objects, except that in this case the result gives us the luminosity of the coloured surface as a fraction of the luminosity of the magnesia surface under the same illumination.

The foregoing account will be adequate to show the general method of use of a colorimeter such as that described. We have considered, so far, only the expression of colours in terms of the arbitrary primaries of the instrument. Such results, while adequate for any self-contained series of measurements, are not directly comparable with results obtained on any other instrument or by any other method. Fortunately, it is quite simple to obtain the necessary data for the conversion of measurements made on any trichromatic colorimeter to a standard system of primaries if desired. Such considerations apply, however, to trichromatic colorimeters in general and not specifically to the instrument which it is the purpose of this paper to describe. The reader is referred to a paper by the author entitled "The Transformation of Trichromatic Mixture Data: Algebraic Method," *Trans. Opt. Soc.* 26 (1924–5) 95, for further information on such conversions, and to the paper on "The Geometrical Solution of Colour Mixture Problems," to which reference has already been made, for various examples of problems involving trichromatic data.

Calibration

It only remains to describe the calibration of the essential parts of the apparatus.

It will be obvious that the "Fullolite" lamp employed to provide light for the primaries must be operated under constant conditions. Although the transmission bands of the filters are sufficiently narrow that moderate variations of voltage do not affect the quality of the primaries, the quantitative relationships between the different primaries are affected by any change in the energy distribution of the source. A sensitive voltmeter should be connected across the lamp terminals, and a variable resistance put in series with the supply in order to adjust the lamp voltage to a suitable value. Lamps should be obtained for which the normal operating voltage is less than the supply voltage, in order that the insertion of the variable resistance may not involve under-running the lamps, with consequent loss in available luminosity, particularly of the blue primary.

It is immaterial at what actual efficiency the lamp is operated. The essential thing is that it should be used under conditions identical with those which prevail when taking the standard white readings which determine the relative values of the scale divisions in trichromatic units. The white readings should be taken frequently in any case, but must certainly be taken after every occasion on which any alteration may have been made in the position of the lamp, as, for instance, if the latter has been moved in its socket.

The scales of the sector openings should be checked, as the proportionality between scale reading and quantity of light reaching the field of view may be disturbed by a variety of causes such, for instance, as non-uniformity of the gelatine filters, non-uniformity of surface brightness of the lamp used, vignetting of the pencils of light on their way through the various parts of the apparatus, etc. The scales can be easily calibrated, one at a time, by using filters of similar colour to the primaries, as when determining the luminosity coefficients of the primaries by the method already described. With the red filter, say, in front of the window, the field is matched, by means of the red primary only, for a sufficient number of field brightnesses to enable a calibration curve of the scale to be drawn. The series of field brightnesses is best obtained by insertion in the beam of rotating sectors of suitable effective transmissions. If such a series of sectors is not available, the distance of the lamp from the magnesium oxide surface may be varied. In such a case the surface should be arranged to be normal to the incident light and not to the line of sight as in the general use of the instrument. It is also necessary in this method to take particular care to exclude extraneous light from the screen, and also to avoid ever having the lamp too close to the screen for the inverse square law to hold.

The green and blue scales can be checked separately in the same way.

It is desirable to make separate calibrations when the neutral reducing glass is mounted on the lantern, as irregularities in the properties of the glass may affect the readings. It has already been pointed out that separate white light readings should be taken when the neutral glass is in use, as the factors for converting scale readings to trichromatic quantity units which apply in the absence of the filter may not apply when it is fitted.

The experimental instrument herein illustrated was made in the Optics Workshop of the National Physical Laboratory. An instrument is also being constructed by Messrs Adam Hilger, Ltd., who propose to add this colorimeter to the list of instruments which they manufacture.

Appendix I

A White Light for Colorimetry

The provision of a standard white light for colorimetric work is not, unfortunately, a straightforward matter. In computation work it has become customary to adopt, as the standard energy distribution for white light, that of a black body at a temperature of 5000° K. Such a source is not obtainable in the laboratory. Although light similar in colour to that corresponding to even higher black body temperatures than 5000° K. is emitted by some high current density arcs, the radiation from such sources departs very considerably from black body distribution. Moreover, such sources are neither steady enough nor reproducible enough for use as standards of illumination.

It is necessary, therefore, to obtain the light from a source of much lower temperature, using a bluish filter to alter the energy distribution as nearly as possible to that of a 5000° black body.

Considerable work has been devoted at this laboratory to developing a satisfactory source of white light. The most promising substances for use in a filter to raise the colour temperature of a low temperature source were first, apparently, discovered by Prof. Pfund. The necessary blueness can be obtained by a solution of ammoniated copper sulphate, but the absorption curve of this substance departs considerably from that required to give a black body distribution. Prof. Pfund found that the absorption properties of an aqueous solution of copper sulphate and cobalt sulphate were of such a nature as to correct a large proportion of the outstanding defects of the ammoniated copper sulphate. The filter should, therefore, consist of two cells of colourless optical glass, one of which contains the ammoniated copper sulphate and the other the solution of copper and cobalt salts. Pfund has not, however, published particulars of the proportions of these various elements of the compound filter, and an enquiry by the author shortly after hearing of the filter elicited the reply that Prof. Pfund had not completed his investigations. As the matter subsequently become one of urgency, the author's colleague, Dr A. F. A. Young, started to investigate the filter *ab initio*. The investigation is not yet completed, unexpected difficulties having been encountered in the final stages, owing to causes which cannot be discussed here. It may be useful, however, to give provisional data at this juncture, as there is not at present available any satisfactory illuminant at all.

It is not possible with any combination of these substances to get an accurate black body distribution in the filtered light. The departures from the ideal are naturally smaller the less concentrated the solutions; hence it is desirable to start off from as high an initial temperature as is practicable. A gas-filled lamp operated at a fairly high efficiency was regarded as the most suitable of the available sources, and the white light filter which is being sought is one which will raise the apparent colour temperature of a source of this character from 3000° to 5000° K.

This presents us with an auxiliary problem in connexion with the initial source. Gas-filled lamps are not yet suitable for standards of colour temperature, neither their constancy of behaviour nor the accuracy with which the relationships between efficiency and colour temperature are known for them, particularly when run at high efficiencies, being adequate.

The region of colour temperature for which the most reliable data exist is that obtainable from vacuum metal-filament lamps at approximately normal efficiencies. A point of particular interest within this range is 2360° K. which may also be obtained from an acetylene flame using the Eastman-Kodak standard burner.

A second filter is therefore being worked out which will raise the apparent colour temperature of a source at 2360° K. to 3000° K. This filter, when placed between a source at 2360° K. and the screen of a photometer, will give to the latter the colour corresponding to 3000° K. A gas-filled lamp is arranged on the other side of the photometer and its voltage adjusted until a colour match is obtained,

when it will be operating at a colour temperature of 3000° K. The lamp may then be used at this voltage as a sub-standard of the colour temperature 3000° K., to serve as the basis of the white light. Needless to say, the voltage at which it is to be operated must be frequently revised by matching against the initial standard source at 2360° K., as the ageing of the sub-standard is fairly rapid when run at such a temperature. It is necessary to procure lamps with a normal operating voltage considerably under the supply voltage, otherwise it may be impossible to reach the desired colour temperature. Low-voltage lamps with thick filaments are more suitable for running at such a colour temperature than high-voltage lamps of small current consumption.

Approximately correct filters for use as above are

2360° to 3000° K.:	Solution A:	Copper sulphate	1·41 gm.
		Ammonia (density ·90)	141 c.c.
		Distilled water to	1000 c.c.
	Solution B:	Copper sulphate	11 gm.
		Cobalt sulphate	8·5 gm.
		Distilled water to	1000 c.c.

Solution A and solution B are contained in separate compartments of a double cell of which the walls are of one of the white optical glasses (say hard crown). The thickness of each solution should be 10·00 mm.

3000° to 5000° K.:	Solution A:	Copper sulphate	2·30 gm.
		Ammonia (density ·90)	230 c.c.
		Distilled water to	1000 c.c.
	Solution B:	Copper sulphate	13 gm.
		Cobalt sulphate	12 gm.
		Distilled water to	1000 c.c.

A similar cell should be used as in the case of the other filter.

These solutions will enable a much closer approach to a standard white light to be obtained than any of the filters at present available. It is emphasised, however, that these are provisional figures only, and that modifications must be looked for in a subsequent publication, when full particulars of the actual energy distribution, etc. will be given. It can be confidently stated, however, that such modifications will not involve any serious difference in the quality of the light.

Appendix II

Conditions of Illumination for Colour Standardisation

In the case of colour produced by transmission through glasses or other transparent materials, there is, as far as I am aware, no divergence either of opinion or of practice as to the conditions under which colour should be measured. Such a filter is always mounted approximately normally to the light rays which ultimately reach the eye.

In the case of opaque materials there is no general concensus of opinion or practice despite the fact that the colour of many such materials depends to a marked extent both on the distribution of the incident illumination and on the direction of observation.

It is clear that for purposes of measurement some definite set of conditions must be standardised. The author has made experiments with various arrangements, including that in which a whitened sphere is employed to supply illumination in all directions, as employed in an apparatus described by I. G. Priest*.

The whitened sphere was ultimately discarded as unsatisfactory for the following reasons. It puts serious practical limitations on the type of lamp used to illuminate the specimen. A small type of lamp must be used unless the sphere is of inconveniently large dimensions. More important, however, is the fact that any selectivity in the reflecting properties of the whitening material is amplified owing to the fact that most of the light reaching the specimen has had several reflections at the walls of the sphere.

Further, with this method it would be very difficult, if not practically impossible, to have the *incident* light of the standard white. It is necessary, instead, to put the filter in the reflected beam after emergence from the sphere. In the large majority of cases this is immaterial, the resultant colour being the same in either case. There are, however, a large number of ordinary dyestuffs which are appreciably fluorescent, and some materials are, of course, fairly strongly fluorescent. In such cases it may not be immaterial whether the filter is in the incident or reflected beam. It therefore seems desirable that for general work the incident light should be of the standard white. Still another difficulty imposed by the whitened sphere arises in those cases in which for special purposes an illuminant other than the standard white may be wanted. The types of light source which may be fitted inside a small sphere are necessarily limited. It is undesirable that the apparatus used for obtaining standard conditions of illumination should not be easily utilised with any source of light which it may be necessary to employ.

In any case, there is no particular virtue in hemispherical illumination from the point of view of colour measurement. The fact that colour does depend on distribution of incident light and direction of observation makes it quite clear that no method of test will give results applicable to all the possible circumstances in which a material may be seen. On the whole, however, the circumstances in which colours are *most often* seen, particularly when under critical observation, are those in which the surface is viewed normally and the *bulk* of the illumination comes from some oblique direction.

Such considerations, combined with those of practical convenience, have led us at the National Physical Laboratory to make colour measurements with specimens normal to the direction of observation, the illumination being confined to a narrow beam incident at 45°. *It is strongly recommended that this procedure be adopted as standard practice in colorimetric work.*

A very convenient piece of apparatus for ensuring the realisation of the suggested

* *Journ. Opt. Soc. Amer.* 8 (1924) 173.

conditions is shown in Fig. 6 and in horizontal section in Fig. 7. It consists of a box of the shape shown, entirely closed except for the end of the oblique portion and two circular holes, *A* and *B*, each of 5 cm. diameter. The box is so mounted in relation to any colorimeter with which it is to be used that the centres of the two apertures are approximately on the line of sight of the instrument. If the colorimeter is one which is fitted with a lens window to throw structural details of test

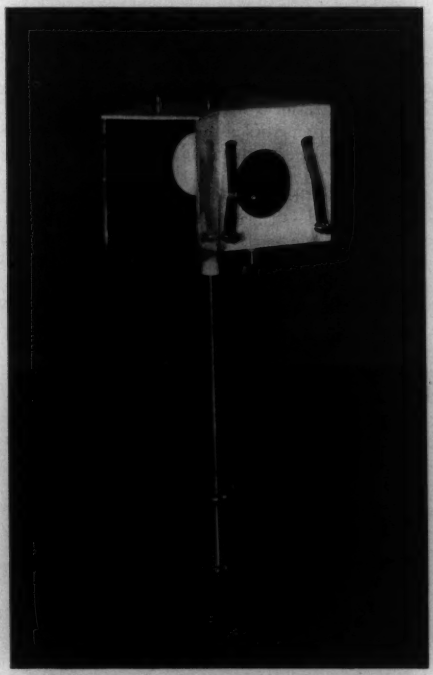

Fig. 6

surfaces out of focus (as in the case of the colorimeter described in the present paper), the aperture *B* should be in or near the focal plane of this lens. Specimens for test by reflected light are placed against the outside of the box so as to cover the aperture *B*. Clips are provided for holding small pieces of material in position. Large specimens must be held against the box by any convenient means.

Light reaches the surface of the specimen, at an inclination of approximately 45°, from the standard illuminant (or whatever illuminant may for particular purposes be in use), which is mounted at a suitable distance along the axis of the oblique portion of the box. If the standard white light is in use, the compound filter described in the previous Appendix may be placed at *F*. The interior of the box is coated a dull black to prevent any appreciable amount of scattered light reaching the specimen, and, if need be, external screens should be arranged to ensure that only light from the desired source can enter the box.

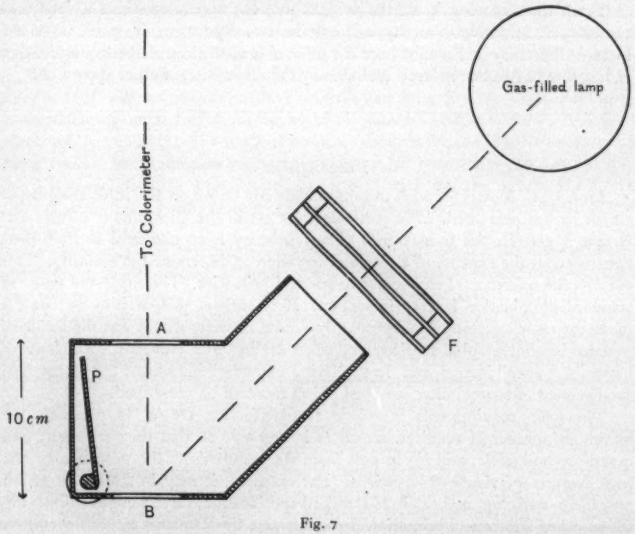

Fig. 7

A rectangular plate *P* is attached to a vertical spindle which passes through a bearing in the bottom of the box. A milled head is attached to the spindle by which the plate may be swung round from the position shown in the figure until it lies flat against the back wall, covering the hole *B* and receiving the incident light on its interior surface, which is coated with magnesium oxide.

Thus, by swinging *P* into or out of the incident light, we obtain, in the field of the colorimeter, either the standard white or the colour under test. To render examination and maintenance of the white surface as convenient as possible, the top of the box is held in position by two milled-head screws, so that it may be easily removed. The plate *P* may then be slipped off the spindle for examination of its condition and recoated with magnesium oxide when required. The ease with which

this can be done renders it unnecessary ever to use the screen in a questionable condition.

When measuring a transparent colour, the white screen is put in position and the filter to be tested is placed in front of the aperture *A*, or in any convenient position between that and the colorimeter.

In measuring the colour of the light given by a source other than the standard white, the latter must be removed and the source in question employed in its stead to illuminate the magnesia screen.

From the foregoing it will be evident that the apparatus forms a very convenient unit for use with any type of colorimeter. By turning it upside down the incident light may be brought from the other side of the line of observation to that shown should this be rendered desirable by the disposition of other apparatus.

DISCUSSION

Prof. L. C. Martin: I wish to congratulate Mr Guild on the interesting paper which he has presented. The particular interest in the trichromatic colorimeter is that it permits the transference of any primary from one field to the other, apparently at the expense of a single observation to determine the quantity. This permits the reference of any colour to spectral primaries. The suggested mode of standard illumination is very important. It is curious that, writing in 1922, I made the same suggestion independently—quite unaware that it was the standard practice in photographic sensitometry when dealing with reflection coefficients of "bromide paper" images, etc. In specifying a standard illuminant the geometrical arrangement of source, filter, and test object must all be prescribed.

The main requirements in colour standardisation are (*a*) to prescribe the minimum number of required constants in any way so that they are easily and exactly reproducible, and (*b*) to pass from the standards to the unknown by the least number of visual observations. For example, it may in future be found possible to make up, accurately to standard specification, three wedges embodying the so-called subtractive primaries. In such a case the standards would depend on quite arbitrary physical things, and this applies also to the standard source of light which would still be necessary. The step from the standards to the unknown would then, in general, involve only one visual colour match. Of course, if it is not possible to make a wedge conform accurately to a standard, calibration is necessary, and no advantage would be gained unless the calibration could be conducted by physical means; but if it is possible to prescribe a particular chemical solution as a standard means of correcting the colour of the standard illuminant, it *may* be possible to evolve a solution with which to fill a standard wedge. It is a matter for experiment.

Mr Guild: I am indebted to Prof. Martin for his interesting comments. It may be well to emphasise that where an auxiliary measurement can readily be

made with an accuracy much greater than that required, it does not constitute an additional observation in the ordinary sense, inasmuch as its contribution to the probable error of the result is negligible. This applies to the evaluation of the amount of any primary transferred to the test side of the field when measuring highly saturated colours. The amount so transferred is always only a small contribution (in chromatic value) to the total colour which is matched. Errors in its evaluation are proportionately reduced in their effect on the final specification. Moreover, when evaluating the added primary, this is alone in the field, and is matched by a setting of one primary only—a purely photometric match of two fields of identical colour which can be performed with quite a high accuracy relative to the ultimate requirements.

As regards the suggested standard method of illumination described in Appendix II, I was not aware that Prof. Martin had suggested the same conditions until about a year ago when he did so in his Cantor Lectures (*Journ. Roy. Soc. Arts*, vol. 73, p. 233). The method had then been in use at this Laboratory for some time. I was particularly gratified, however, to find the suggestion put forward independently by Prof. Martin, as it indicated that the advantages of the system, as compared with more complicated arrangements, were evident to others besides myself.

With reference to the possibilities of a subtractive system for ultimate standardisation, it can be said at once that no simplification, but greatly increased complexity as compared with the additive method would necessarily be involved. It is not enough to have accurately reproducible filters and light sources to define your primaries—this could be done just as easily for an additive as for a subtractive instrument, were any advantage likely to accrue which would compensate for the disadvantage of using liquid filters as components of an instrument. It is necessary to evaluate the units in which these primaries are to be expressed in terms of visual relationships such as matches of white, etc., otherwise a purely arbitrary system results which, however reproducible, is entirely useless, as no colour relationships can be worked out in terms of the system. It is necessary to correlate measurements by any system with the fundamental data of the trichromatic colour chart in order that the results may be comparable with one another and with the physical analysis of the stimulus.

The correlation of a subtractive system with such fundamental data is very much more complicated than is the case with additive systems. Further, the subtractive system suffers even more than the additive system as regards inability to match saturated colours. In order to make the method universal, a process similar to that employed in the trichromatic colorimeter for modifying saturated colours until a match is possible would have to be introduced. The evaluation of the effect of such modification, while very easy in the case of the additive method, would be much more difficult by the subtractive method.

A CRITICISM OF THE MONOCHROMATIC-PLUS-WHITE METHOD OF COLORIMETRY

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ABSTRACT. The author contends that, from the practical standpoint of quantitative colorimetry, hue and saturation are not the fundamental elements of colour quality but must be regarded merely as derivatives of the trichromatic constitution of the colour. Practical and theoretical reasons are advanced to show that whatever may be the advantage of specifying colour quality by hue and saturation there are grave objections to practical methods of colorimetry involving the direct measurement of saturation.

One of the most favoured methods of colour specification is that in which the three elements Hue, Saturation, and Luminosity are treated as the variables. The last of these relates to the total amount of light reaching the eye. Colours which differ in luminosity only may be made to look alike by merely increasing or diminishing the amount of light from one of them without altering the distribution of energy. It is the first two variables, hue and saturation, which determine the quality of the colour. Colours of different quality cannot be made to look alike by any adjustment of their relative illumination.

This system of specification is based on the law, apparently propounded by Grassmann*, that any colour, no matter how it is composed, must have the same appearance as the mixture of a certain saturated colour with white. The saturated colours are those of the spectrum and the range of non-spectral colours obtainable by mixing the extreme red and violet of the spectrum in varying proportions (the purples). For colorimetric purposes all colours which only differ in the proportion of white in their composition are regarded as having the same hue†.

Thus all the colours which may be matched by mixing white with, say, monochromatic light of wave-length λ have the hue defined by this wave-length, and differ only in saturation.

The saturated colours of non-spectral hue, being themselves essentially mixtures,

* According to Helmholtz, *Physiological Optics*, English Translation, vol. II (1924) p. 133.

† It is by no means certain that this correctly corresponds to the subjective sensation of hue. There are positive statements in the literature that the apparent hue of colours may change on diluting them with white. In some of these experiments the results are invalidated by the "white" employed not being white at all. Whether this applies to all of such observations I am unable to say. In any case for the purposes of colorimetric definition we take as the hue of a colour the hue of the saturated colour which would be left if the white were abstracted.

are not readily defined by a simple physical characteristic such as wave-length, nor are they readily produced in full saturation. It is, therefore, impracticable to use them as the basis of a definition of hue and saturation. Colours in this region are, however, complementary to the spectrum colours, and we can define the hue of any such colour by the statement that it is *complementary* to a certain spectral hue, while its saturation can be measured in terms of the proportions of the colour and its spectral complementary which match white.

On this system, therefore, all colours are specified in terms of some monochromatic radiation and white, and the system is usually referred to as the "monochromatic-plus-white" system. The proportion of monochromatic and white constituents in the colour, which determines the saturation, is invariably expressed on a *luminosity* basis, and not in quantity units defined by colour-match relationships.

The relationships between the monochromatic-plus-white system and the trichromatic system have been explained by various authors. In the form in which I prefer to have them, they will be found in a paper recently contributed to this Society on "The Geometrical Solution of Colour Mixture Problems*." The reader is referred to this paper for a complete treatment of those relations between colours which determine the results of colour matching. *A knowledge of the contents of the paper will be assumed in what follows.*

A knowledge of the hue and saturation of a colour as it appears to the normal eye enables us to locate the point which represents this colour on the trichromatic colour chart, and vice versa.

In fact any method of colorimetry which gives results that can be reduced eventually to a position on the colour chart may be employed to obtain measurements of hue and saturation.

The accuracy of such conversions depends of course on the accuracy of the data embodied in the colour chart, but so, also, does the relationship of the hue and saturation of a colour to the physical constitution of the stimulus. Unless we regard a system of colour specification as something entirely self-contained, and are prepared to leave out of account its relationships to other systems and to the physical properties of the stimulus, we are dependent for the ultimate accuracy with which a colour can be defined on the accuracy of the fundamental visual data contained in the mixture curves of the spectrum employed in constructing the trichromatic colour chart. We cannot calculate the results of colour mixture except in terms of these data. In fact all fundamental methods of colour specification, whether explicitly trichromatic or not, implicitly involve these data whenever we do more than consider isolated colours by themselves. Colorimetry being essentially a measurement of the *relationships* between colours, the expression of results is essentially an expression of such relationships, and we cannot, by merely choosing a certain form of words to designate a colour, escape from the fact that what we are dealing with is fundamentally a trichromatic equation, or its equivalent, a position on the colour chart.

* *Trans. Opt. Soc.* 26 (1924–25) 139.

Thus, while hue and saturation have a unique physiological or psychological significance in that they are the visually apparent attributes of a colour, when we come to regard them from the quantitative point of view of colour measurement and specification they are not fundamental, and can only be regarded as derived functions of the trichromatic constituents. From this standpoint we must consider the monochromatic-plus-white or any other method of specification merely as methods of defining the position of a colour on the chart, and we must consider methods of colour measurement merely as methods of finding the position on the chart, no matter in what derived form we may ultimately wish to express the results.

If the chart were an accurate representation of the properties of the normal eye, all theoretically sound methods of measurement would arrive at the same result. Inasmuch as the best existing data are inaccurate, different methods give different results, but no one of these is entitled to be given preference, since they each depend on different, but equally fundamental, relationships among the constants of vision, all of which would be true and mutually consistent for accurate data.

Thus, when we have arrived by any practical method which is theoretically sound at a position on the chart for any colour, we have said all we can say about that colour in the present state of our knowledge regarding the properties of the eye. The fact that another equally sound method may lead us to an appreciably different result is unfortunate, but is unavoidable till a better set of trichromatic mixture curves is obtained. We cannot escape the difficulty by giving arbitrary preference to one method of measurement, since any system will break down when applied to the relationships between different colours unless the fundamental data are accurate.

This is emphasised because there appears to be a tendency to assume that the adoption of the monochromatic-plus-white system of colour specification, and the use of methods of measurement which arrive at the elements of this specification directly, will short-circuit the troubles arising from the inadequacy of the existing trichromatic data and will evade the necessity of a modern revision of these data—a process which has hitherto presented very considerable practical difficulties.

I purposely wish to avoid in this paper any discussion of the relative merits of the monochromatic-plus-white, trichromatic, or any other system as a basis of specification. From this point of view they must all be regarded as mathematically equivalent methods of expressing the same essential fact.

The practical problem of colorimetry is that of arriving at the essential fact, and is not affected in principle by the choice of one or other of the possible methods of expressing the result of the measurement.

Colorimeters for measuring hue and saturation directly have been devised by P. G. Nutting* and by I. G. Priest†. An improved form of Nutting's instrument is made by Messrs Adam Hilger, London.

These are constructed on entirely different lines, Priest's colorimeter, which is

* *Bull. Bur. Stand.* 9 (1913) 1.

† *Journ. Opt. Soc. Amer.* 8 (1924) 173.

really an assembly of apparatus rather than an instrument properly so-called, having, in the writer's view, many advantages over the other: but the principles of operation are identical in both; the colour under test, if of spectral hue, is matched by a mixture of monochromatic light (obtained spectroscopically) with white. The relative proportions, by luminosity, of the two constituents are deduced from a subsequent measurement in which they are compared with a flicker photometer.

If the colour under test is of non-spectral hue (the purple group) it is itself mixed with monochromatic light to match white. The proportions of the mixture are subsequently determined by a flicker photometric comparison of the test colour with the monochromatic constituent.

I wish to point out both practical and theoretical objections to the introduction of a *photometric* process in a determination of *colour quality*.

The practical objection arises from the inherent difficulties of heterochromatic photometry. The only method of reasonable precision is that employed in these two instruments, namely, the flicker method. This method, however, only gives results approximating to a true comparison of luminosities when the field brightness is high, 25 metre-candles being specified for standard practice. With the Nutting colorimeter it is certainly not feasible to obtain a field brightness of 25 metre-candles except when the hue is near the brightest part of the spectrum. For hues near either extremity, the flicker match has to be made with much lower illuminations and consequently does not give true relative luminosity. The same difficulty must exist to a greater or less extent with any apparatus of this kind. The fact that flicker measurements cannot be carried to the extremities of the spectrum* makes it clear that attempts to measure saturation directly must break down for colours of hue near either end of the spectrum. Apart from this, in the case of colours of non-spectral hue, the test colour itself is one of the components in the flicker match. Since any colour under test may be of low luminosity irrespective of its hue, it may happen with any colour of the purple group that the photometric measurement has to be made under conditions of luminosity for which it cannot give the correct result.

It may not be impossible to overcome such difficulties, though no practicable solution has presented itself to the author, but at least it is obvious that to effect a satisfactory measurement of saturation in all cases which may arise in practice would be an extremely difficult matter.

The theoretical objection to this method of colorimetry, which would still exist were there no practical difficulties involved in the measurement of saturation, arises from the essential difference in type of the sensations of luminosity and colour quality. Although these sensations are intimately related, the relationship is not unique, either for the same individual under all circumstances or for different individuals. From any sufficiently large number of observers we can obtain a "normal" luminosity function, as represented by the standard visibility data, and

* Both ends of the accepted visibility curve are based on measurements made by other methods, the flicker data being inadequate for these regions.

a "normal" chromatic function, as represented by a set of trichromatic mixture curves for selected primaries. These two "normal" functions have a quite definite relationship which we can express by assigning luminosity coefficients to the primaries of the trichromatic system. Such a standardised relationship for the "normal" eye may be employed to calculate the hue and saturation of a colour known by its position on the colour chart, and the result will be true for the normal eye. Such a procedure is unobjectionable, inasmuch as the use of standard transformation constants makes the result, in effect, merely an alternative statement of the trichromatic data.

A different position arises when the relationship between the luminosity and chromatic functions of *any particular individual* enters into the measurement of a colour.

An enormous amount of controversial opinion has been expressed on the extent to which the two functions may vary independently. Some investigators of the statistics of vision assert that quite large departures from normality in one function may exist without any appreciable abnormality in the other, a statement which is categorically denied by other experts. However this may be as regards large abnormalities, there is no reason to doubt that small abnormalities in either function, of the magnitude which may be found among persons possessing no marked defect of vision, may occur independently.

There is a vast variety of abnormalities of colour vision. It would be impossible to discuss the effects of all possible types on both the chromatic and luminosity functions, but consideration of one type will serve to illustrate the point at issue. There are *some* abnormalities which can be explained as due to a diminution of the sensitivity of one element of the tri-receptor process as compared with the other two, without modification of the relative extent to which it is stimulated by light of different wave-lengths. The *shape* of the curve representing the contribution of this element to the luminosity of the spectrum colours is normal but the ordinates are smaller. A moment's consideration will show that in such a case the trichromatic mixture curves of the spectrum, *expressed as usual in units based on the stimulation by white*, and the colour chart derived from these, will be the same as for the normal eye. Such an individual will make normal colour matches. It is true that colours must appear differently to him than to a normal individual because of the difference in relative stimulation of the three elements, but in colorimetry we are never concerned with the actual appearance of a colour but with identity of appearance of two colours—one the unknown colour and the other an identical colour synthesised from known constituents, or otherwise suitable as a standard. Thus a person might be quite normal in all colour-match relationships and be a quite suitable individual for colorimetric work and yet be distinctly abnormal in what he actually sees, and also in his luminosity relationships, which can only be normal if all three elements are normally stimulated.

Such a person would obtain the correct (normal) position for a colour on the chart as the result of any method of measurement involving colour matches only, as, for example, with a trichromatic colorimeter, but would make an abnormal

determination of saturation from a photometric measurement. Any abnormalities of vision which are not exactly of the type considered but approximate to it will in general give rise to relatively large changes in the visibility function and relatively small changes in the colour-matching properties of the eye. Persons so affected would make much larger errors in a colour measurement involving a photometric determination of saturation than they would when employing methods depending on colour matches only.

An examination of the individual visibility curves obtained by any of the investigators who have examined numbers of people shows quite clearly that comparatively enormous differences in the relative visibility of colours in those parts of the spectrum where the sides of the visibility curve are steepest occur for people who cannot be regarded as seriously abnormal in their colour vision.

Further, apart from abnormalities in different individuals, the photometric properties of any eye vary more seriously with field brightness (Purkinje effect), or with fatigue, than do the colour-matching properties, for the reason already pointed out, that the latter properties are relatively little affected by any cause which is wholly or approximately of the nature of a variation in relative sensibility of the elements of the tri-receptor process. Thus, even for an observer who is normal in both the chromatic and luminosity functions, serious errors can arise in measurements of saturation in circumstances which would only slightly if at all affect his colour matching.

There is still another fundamental objection to the method. Waiving the points already dealt with, we have to consider whether the photometric criterion is adequately sensitive in all cases. Leaving out of account all practical and theoretical difficulties in the way of measuring saturation, is it possible to locate a point in the colour chart by a photometric measurement sufficiently closely that the error cannot exceed the amount which would be detectable as a colour difference?

I have never seen this point discussed, and there are not sufficient data regarding the liminal values at different regions of the chart to investigate it in any detail. It is possible to state, however, that with some colour mixtures a change corresponding to a difference of two per cent. in the photometric comparison of the constituents would be appreciable as a colour difference. Even smaller tolerances may exist at other parts of the chart.

It seems clear from the foregoing discussion that it is unsound to determine colour quality by methods involving a photometric measurement, and that such determinations should involve only those properties of the eye which are directly concerned in colour-match relationships.

Thus, whatever may be the merits of the monochromatic-plus-white system for colour *specification*, the attempt to embody the system in a method of colorimetry is subject to grave objections.

The only practical advantage which colorimeters involving this principle possess is their suitability for dealing with highly saturated colours of spectral hue. It is very convenient for certain purposes to employ a method of colorimetry involving the use of monochromatic light as one of the constituents of the matching

colour. In another paper I describe a new method of colorimetry in which monochromatic light is employed but which does not possess the disadvantages of the monochromatic-plus-white method*.

DISCUSSION

Prof. L. C. Martin: The "wave-length of dominant hue," "purity," and "relative brightness" specification of colour is, of course, the readiest road to a mental picture of the result, and this probably accounts for the preference which the monochromatic analysis has been given. I quite agree with Mr Guild that there is no fundamental advantage about the process, but I have not sufficient practical experience of abnormal colour vision to say whether the luminosity function is relatively so uncertain as he suggests. My experience with the Nagel anomaloscope, in which red + green matches yellow, suggests that there are very few people who agree in the exact proportions of the red and green required. One well-attested statement about a large class of colour-blind subjects is that they will *agree* to normal colour matches although they will not repeat them when allowed a free setting.

I incline rather to think that colorimetry may in the end prove to be a very useful affair for individuals, but unsatisfactory in the general sense because the risks involved would be too great from the commercial standpoint.

Mr Guild: There are two statements regularly made about hue and saturation (or its inverse "purity"), firstly, that these are the visually apparent attributes of a colour, and secondly, that a specification of these quantities is, in Prof. Martin's words, "the readiest road to a mental picture of the result." Both statements appear to be regarded as so obvious that they may be taken for granted. The first is, I think, as nearly true as need be, though some recent enquiries from persons having no colour education revealed some surprising views as to what is or is not "apparent" in a colour. The second statement seems to me to be quite unjustified. Neither a statement of wave-length and purity nor of trichromatic constitution conveys any meaning to a person who has not learnt the language involved. Persons who have not only learnt the language but have had a very wide experience of either method of specification will form a fairly close idea of the colour indicated by a given specification on either system. The relative advantages must be judged by the accuracy of the impression formed by persons who know the significance of the terms but who have not a sufficiently wide experience to conceive the corresponding colours as the result of education rather than as mental interpretation of the elements of the specification.

* *Trans. Opt. Soc.* 27 (1925–26) 139.

I agree that many such persons will have less hesitation in forming a mental picture of a colour from a statement of hue and purity than from a statement of trichromatic constitution, but will the mental picture they form be any nearer the truth, or as near it? My own experience leads to the conclusion that there is no justification for claiming any advantage for the "monochromatic" specification in this respect. It requires a very experienced colorimetrist indeed to form an approximately correct idea of the appearance of a colour of, say, hue ·44μ and saturation 50 per cent. I am certain that for such a colour a trichromatic specification would lead to a much closer guess at the actual appearance by the great majority of colorimetrists.

This, however, while of interest in other connexions, is not the issue in the present paper, which deals with the drawbacks to practical methods of colorimetry involving heterochromatic photometry on the part of the colorimetrist. The statements made on the possible independent variations from normality of the luminosity function and the chromatic function in different eyes are not made on my own authority but are based on such evidence as is obtainable from a study of the literature of the subject. While the evidence as regards large independent variations is conflicting, there is no room for doubt as regards the possibility of small but appreciable variations. Prof. Martin, himself, in his Cantor Lectures (*Journ. Roy. Soc. Arts*, vol. 73, p. 233) quotes evidence that there are "considerable differences in the 'luminosity curves' for different 'normal' observers under similar conditions, although corresponding differences are not found to appear in the primary stimulation curves."

Apart from this, the possibility of defects of vision of such a character that colour-match relationships remain unaffected or only very slightly affected, although both the appearance of colours and the luminosity relationships are quite seriously abnormal, cannot be disputed. In cases of this type in which the abnormality is great, i.e. in which one of the fundamental primaries is very little stimulated, there is a loss in sensitivity which might result in a match being made by the subject which would appear wrong to a normal observer, but this is due merely to the wider limits of tolerance for the subject in question and not to an actual change in the colour-matching relationships. The important point is that in cases of this kind which are not sufficiently pronounced for the loss of sensitivity to disqualify the subject from colorimetric work, the change in the luminosity function may nevertheless be sufficient to invalidate his measurements of saturation, whereas, if he employs methods of a purely colour-matching character, he may be a quite satisfactory observer.

I wish nothing in this paper to be regarded as committing me to the view that any pronounced cases of colour blindness can be regarded as due *entirely* to difference from normal in degree of stimulation of the primaries of the Young-Helmholtz theory. I have not adequate personal knowledge of the evidence to form an opinion one way or the other; but that many cases exist which partake, at least approximately, of such a character is indisputable, and is enough to substantiate the thesis of this paper.

As regards variations in any one eye due to abnormal conditions of fatigue, luminosity, etc., these can be verified by anyone. A match made in any suitable colorimeter will remain matched, at least very closely, even after the eye has been subjected to an amount of differential fatigue sufficient to produce a marked change in the appearance of the colour and to disturb the luminosity function to a marked degree. Thus, even for an eye normal in every respect, much closer limits must be set in the experimental conditions for measurements in which the luminosity function is to be involved than for those involving colour matching alone. This in itself would be a serious objection to the monochromatic-plus-white method, even if the other criticisms were invalid.

ON A NEW METHOD OF COLORIMETRY

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ABSTRACT. The paper describes a new method of determining the quality of a colour, as defined by its position on the trichromatic colour chart, in which only the colour-matching properties of the eye are involved. The determination depends on two colour matches in each of which the test colour is matched by a mixture, in unknown proportions, of a standard colour and a monochromatic colour obtained spectroscopically, a different standard colour being used for the two matches.

An instrument is described in which the method is embodied.

In another paper* the author points out certain disadvantages attached to the use of colorimeters of the monochromatic-plus-white type, in which hue and saturation are directly determined.

Such instruments have, however, advantages for some classes of work because they utilise spectrum colours, and the method of colorimetry now to be described enables this advantage to be retained.

The fact that many colours can be matched by a mixture of monochromatic light with white, while the remaining possible colours are complementary to those of the spectrum, which is the basis of the monochromatic-plus-white system, is always stated as though some peculiar property of white were involved, and there is no doubt that the law is regarded by many as having such a significance. It is, however, only a special case of a general law which we may express in the following form.

With respect to any colour X, all other colours fall into either of two groups. Any member of group I can be matched by mixing X with monochromatic light of suitable wave-length. Any member of group II is complementary, *with respect to* X, to some spectrum colour; that is to say, it can be mixed with monochromatic light so as to match X.

The above law is perfectly general and applies whatever colour, either saturated or unsaturated, X may be. Grassmann's law, on which the monochromatic-plus-white system is based, is merely the special case of the above law in which white functions as the colour X, and involves no special property of white.

The general law follows at once from the elementary properties of the colour chart† and need not be elaborated.

* *Trans. Opt. Soc.* 27 (1925–26) 130.

† In what follows a knowledge will be assumed of the contents of a paper by the present author, "The Geometrical Solution of Colour Mixture Problems," *Trans. Opt. Soc.* 26 (1924–25) 139.

We could, therefore, base a system of colour specification or measurement on identical lines with the monochromatic-plus-white system, but using any colour we chose as the fixed colour instead of white. Some fixed colours would be inferior to white inasmuch as a greater proportion of the colour field would require to be dealt with on the complementary basis, but others would be better than white in this respect.

Fig. 1 is the colour chart based on the Koenig-Ives "Fundamental Sensations" as primaries. The colour field is bounded by the curved spectrum locus and the dot-dash line which joins its extremities. Let X be chosen as our fixed colour, while r' and v' are the spectrum colours complementary (with respect to X) to the red and violet ends of the spectrum respectively. Then all colours to the right of $v'Xr'$ fall in group I only; all colours to the left of rXv fall in group II only; all colours in the regions rXv' and vXr' come in both groups. It is evident that the more nearly X is situated to the boundary rv, the smaller does the complementary

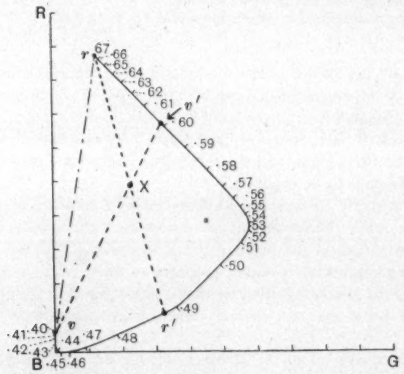

Fig. 1

region become, while if X is located anywhere on rv the whole colour field comes into group I. Of the range of colours represented by rv, the terminal colours r and v are of particular significance, as they are spectrum colours and are uniquely specified by their wave-lengths. Of the two, much the more important is r, the terminal red of the spectrum. The spectrum locus on the chart ends quite definitely at a point, which represents the colour produced by a monochromatic stimulus of wave-length ·67μ *or greater*, there being no change in hue of the spectrum beyond ·67μ for the normal eye. There is thus a monochromatic patch, using "monochromatic" in its correct sense, between ·67μ and the end of the spectrum. This is of great practical importance as it provides us with a perfectly definite colour, obtainable in plentiful quantity and perfect purity by the simplest possible means.

There are various filters available which transmit only this terminal region. With such a filter and any source of continuous spectrum we can obtain with certainty the precise colour r (Fig. 1). This colour, if used instead of white in instruments constructed for monochromatic colorimetry, would effect considerable simplification in the design, for all colours can be matched by mixtures of it with some other monochromatic radiation and no provision would require to be made for measurements by the complementary method.

Thus we might measure a colour by finding the wave-length of the monochromatic light which when mixed with terminal red matches the colour, and subsequently determine photometrically the proportions of the two constituents, exactly as in the ordinary use of the monochromatic method. The result, which would be, of course, merely a new way of finding the position of the colour on the colour chart, could be converted to the trichromatic basis or to hue and saturation as desired.

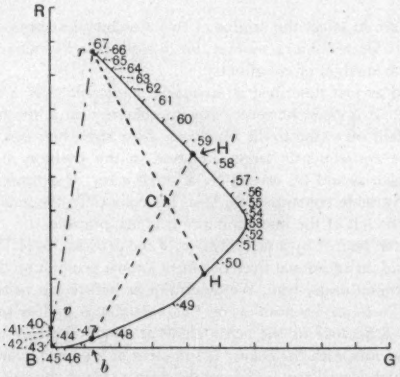

Fig. 2

Such a procedure would, however, be subject to the same objections as I have urged against the ordinary monochromatic method, owing to the introduction of a photometric process into a determination of colour quality. We can obviate this by the following procedure.

Let C (Fig. 2) be the colour to be tested. By trial we find that the monochromatic light H can be mixed with r to match C. This tells us that the colour lies on the line rH. Our problem is to find its position on this line. This can be done by finding another line on which the colour lies, by means of a colour match with a different pair of constituents. The intersection of these lines determines the position of C on the chart. For the fixed constituent of this second match it would be advantageous if we could employ the extreme violet end of the spectrum as in that case

the method would be applicable, in principle, over the whole colour field. We have no definite monochromatic region at this end of the spectrum, however, the hue varying continuously until the luminosity becomes negligible. We cannot therefore use the terminal point, v, of the spectrum locus for our second fixed colour, but have to employ a violet or blue appreciably removed from the end of the spectrum. By means of suitable filters and any lamp giving a continuous spectrum we can obtain fully saturated colours of such hues. Suppose by these or other means we obtain a second fixed colour b (Fig. 2), and find that the colour C can be matched by a mixture of b with monochromatic light H'. This tells us that C lies on bH', and, as we know from the other colour match that it also lies on rH, its position is uniquely determined. It will be noted that we do not have to evaluate the quantitative constitution of either of the two colour matches. All we require to know is that C can be matched by *some* mixture of H and r and also by *some* mixture of H' and b. Thus the process depends wholly on the colour-matching properties of the eye.

As r and b are in effect the origins of two systems of vectors, the intersection of which serve to locate colours, we may, for the sake of a distinctive name, describe this as the vector method of colorimetry.

The method as just described is obviously applicable over a large portion of the colour field. It is clear, however, that for colours near a line rb the sensitivity of the method falls off owing to the smallness of the angle between the intersecting vectors and the consequently large alteration in the position of their point of intersection which would be caused by a small error in determining the wavelengths of the variable constituents. Also, for colours in the small region of the colour field to the left of the line joining rb it is inapplicable.

Such cases can be met by a modification of the process, involving an additional colour match and an additional fixed colour of known position on the chart. Let C (Fig. 3) be the colour under test. We determine as before the vector through r on which it lies. To locate its position on this vector it is useless to determine the vector through b because of the acute angle at which the two would intersect. Suppose now we mix with the colour C sufficient of a known colour g to produce a mixture C' which is well removed from the purple side of the colour field. C' can be easily located in the ordinary way by the two colour matches which determine the r and b vectors on which it lies. The line gC' produced to meet rH determines the position of C. Again it is to be noted that knowledge is not required of the proportions of g and C in the auxiliary mixture. Any mixture of the two serves the purpose. We merely have to choose a mixture which is somewhere in the region of the chart for which the sensitivity of the intersecting vector method is satisfactory.

In practice it is only necessary to apply this additional procedure to purples of fairly high saturation. With this addition, however, the method becomes applicable to all colours.

A Vector Colorimeter

The vector method, as explained above, can be carried out on any colorimeter designed on the monochromatic-plus-white basis without any essential alterations. It is only necessary to insert a suitable red or blue filter in the path of the white beam to obtain the two colour matches required. I have, however, designed a new instrument incorporating not merely the special features of the vector method of colorimetry, but various other improvements on available instruments which experience has shown to be desirable. In both of the only two instruments for monochromatic colorimetry of which I am aware* the colour under test is viewed through the slit of the monochromatic illuminator. This reduces its apparent luminosity to a considerable extent. In the Nutting instrument the light is further reduced by passing through nicol prisms. An unavoidable loss of light from causes

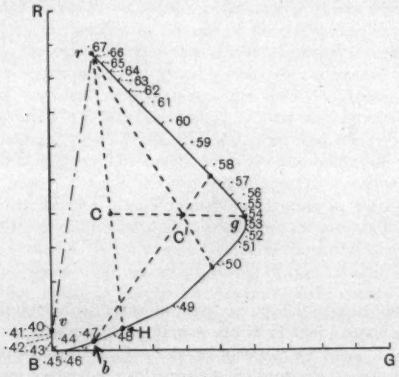

Fig. 3

such as these becomes of serious importance when dealing with colours of low luminosity, e.g. the colours of dark dress materials, etc. Such colours may be rendered practically indistinguishable from black, all sensitivity of measurement being lost. It is essential that such colours should appear, in the field of the colorimeter, of the same order of luminosity as they appear when viewed in the ordinary way, so that the measurement is made on the colour as it would appear to the eye under ordinary conditions. There are practical limitations to the extent to which increased illumination may be used to compensate for loss of light in the instrument, and it is desirable that colorimeters should be so designed that, if need be, the full pupil of the eye may be employed in viewing the test colour,

* The Nutting colorimeter as made by Adam Hilger, Ltd., and the apparatus described by I. G. Priest, *Journ. Opt. Soc. Amer.* 8 (1924) 173.

and that the number of optical parts between the eye and the specimen should be kept as small as possible. These requirements account for most of the novel features of the new instrument, which is shown in diagrammatic plan in Fig. 4. It consists of a Hilger constant deviation spectrometer, standard pattern, to which various attachments have been added. The clamps which ordinarily hold the telescope are removed, and an angle-shaped plate is bolted to the base plate of the spectrometer to carry two collimators C_2 and C_3. The latter takes the place of the usual telescope, but is further removed from the prism. All three collimators are fitted with symmetrically opening slits.

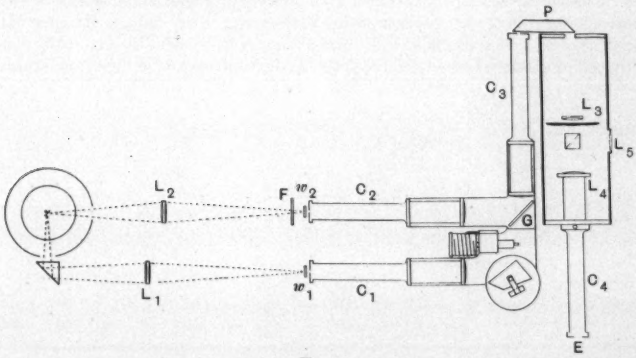

Fig. 4

A "Pointolite" lamp of the 500 c.p. type, in which the luminous source consists of a square plate about $\frac{1}{8}$ inch in width, is arranged in line with one or other of the collimators C_1 or C_2, and, by means of lenses L_1, L_2 and a prism as shown, images of the luminous plate are formed on the two slits. w_1 and w_2 are vertical photometric wedges, ten inches long, maximum density 1·8*, which serve to vary the amount of light reaching the slits.

A plane parallel glass G is mounted to throw a portion of the light from the collimator C_2 into the collimator C_3. Adjustments are provided for G which enable an image of the slit of C_2 to be formed on the slit of C_3. The latter slit therefore receives undispersed light from the collimator C_2 in addition to monochromatic light from the collimator C_1 via the prism of the spectroscope. The wave-length of this constituent may be varied by means of the wave-length drum in the usual way. By placing a suitable red or blue filter at F we may give the light from C_2 the colour corresponding to the point r in the colour chart, or that corresponding to b (Figs. 2 and 3). Thus we have emerging from the slit of C_3 a mixture of either r or b with spectral light of any desired wave-length. The amount of

* As supplied by Messrs Sanger Shepherd.

each constituent in any such mixture may be separately regulated by the photometric wedges.

The mixed light on emergence from the slit of C_3 suffers two total reflections at the cathetal surfaces of a prism P and enters the colour-matching box. In the diagram the prism is drawn with its length horizontal in order that all the essential parts of the apparatus may be shown. In the actual apparatus the prism is vertical, the colour-matching box being above the collimator C_3 instead of at the side of it, as represented in the diagram. The reason for folding the apparatus back on itself in this way, instead of continuing it in a line with C_3, is merely for compactness, and to bring the observer within easy reach of the various controls.

The beam of mixed light, diverging from the slit, meets the lens system L_3, of which more will be said presently, and passes as a collimated beam through the transmitting portion of a photometric cube, being finally refocussed by the lens L_4 to form an image of the slit of C_3 at the ocular pupil E. Light from the specimen under test enters by the window in the side of the box and is directed along the collimator C_4 by the reflecting portions of the photometric cube. This cube consists of two isosceles prisms in optical contact over a central vertical strip of their hypotenuse surfaces. A diaphragm (not shown) containing an aperture 1 cm. square is situated in front of the lens L_4 to limit the field of view.

The field of view is thus square, and is divided into three vertical strips of approximately equal width. The middle strip appears uniformly illuminated by the mixed light from C_3, while the two outer ones are illuminated by the test colour. As this instrument is primarily intended for work in which the test colour comes from a uniformly illuminated area, conditions are favourable for the use of the triple field, although a simple double field is, in the author's opinion, as good for all practical purposes. The reason for having the central strip vertical instead of horizontal is that, owing to the effective aperture of the spectral constituent of the mixed colour being limited to a rectangle (with smaller side horizontal) by the shape of the spectroscope prism, it is easier to get a uniform mixture at all points of a vertical strip than of a horizontal one, the vertical strip being well inside the effective aperture, whereas a horizontal strip would reach nearly all the way across it. The size of the field, which subtends approximately 2° square at the eye, is adopted for reasons discussed in another paper*.

A lens L_5 is mounted as a window. Its purpose is to focus any structural details of the specimen† at the ocular pupil, so that these are not visible in the field of view. This lens is fitted in a mount which can be swung clear of the window, as it is unnecessary to use the lens except when structural details are present.

An iris diaphragm is fitted at E so that the ocular pupil may be adjusted as desired. A pupil of 3 or 4 mm. diameter will be found convenient for general work,

* "A Trichromatic Colorimeter suitable for Standardisation Work," *Trans. Opt. Soc.* 27 (1925–26) 106.

† A suitable piece of apparatus for mounting specimens so that their colour may be measured under standard conditions of illumination and observation is described in an appendix to the paper on a trichromatic colorimeter just quoted, and need not be described here.

but when colours of low luminosity are under test the full pupil of the eye, or a large fraction of it, should be used.

It will be obvious that, while the iris diaphragm fixes the dimensions of the pencils of light reaching the eye from the colour under test, the pencils from the colour-mixing apparatus pass through what is, in effect, a narrow slit at E, namely, the image of the slit of C_3 which is formed at that point. The test and comparison portions of the field are therefore seen by beams which are of quite different cross-section at the eye. This results in parallax effects between the strips, and destroys entirely anything in the nature of a fine and stationary dividing line. One portion of the field makes large excursions relative to the other on moving the head slightly. This effect, which is the result of not looking at the test colour through the slit of the monochromator, is so marked that it would be quite impossible to use the instrument at all were it not possible to overcome it.

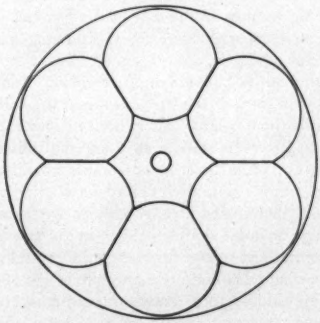

Fig. 5

This is accomplished by the special lens system L_3, consisting of two components, of which one contributes about $\frac{1}{4}$ of the total power. The weaker component is one of a crown of six similar lenses mounted on a glass plate, as shown in Fig. 5. This plate is mounted on an axle and can be rotated by a light belt (a stout thread) from a small motor, causing the lenses to pass rapidly in front of the fixed component. As each lens passes across the beam the slit image at E moves in the same direction as the lens, but at about $\frac{1}{3}$ the rate. As one lens passes out of the beam and the next comes in, another image sweeps across at E. If the rotation is rapid the effect is to spread out the slit image into an area at E which is approximately as broad as it is long and completely fills the largest aperture which it is necessary to use at E. All parts of the field are now seen with identical entrance pupils and parallax effects disappear. Not only so, but several other difficulties which are commonly experienced in practical colorimetry disappear. When two dissimilar colours are superposed in the field of view, the sensation which should correspond

to a homogeneous mixture of the two colours according to the laws of colour mixture is only perceived if the relative strength of the two stimuli is uniform all over the field. If there are local specks or patches in which one or other of the colours is unduly prominent the curious effect arises that the sensations do not blend into a single sensation of the mixed colour, but the two components are separately perceived, one being seen through the other as if a background of one colour had a somewhat coarse veil of the other colour in front of it. The difficulty of getting a convincing colour blend from mixtures of widely dissimilar colours in any apparatus

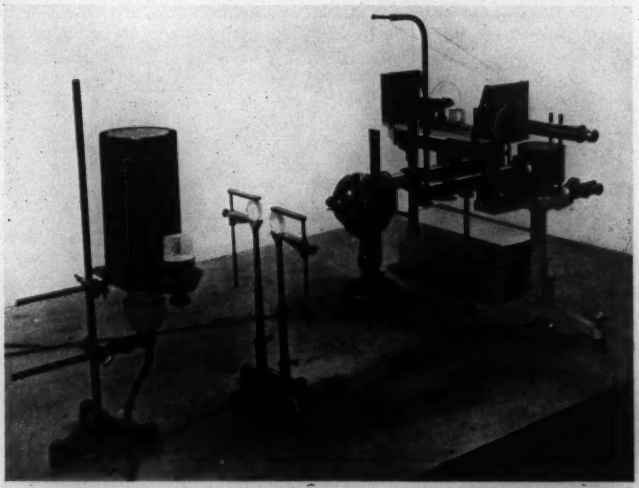

Fig. 6

in which all the parts are stationary is familiar to everyone who has experimented with colour mixing. The effect of striae, bubbles, dust particles, etc. in the parts of the apparatus supplying the constituent colours is to produce local variations in the relative intensity of the constituents at different parts of the field, which give rise to the appearance just described and make accurate matching very difficult. The introduction of moving parts in the optical system, since it evens out all field irregularities, greatly facilitates obtaining a convincing identity of appearance in the test and comparison fields. The actual speed of rotation required is not great since the slit passes across six times for each revolution of the disk.

General views of the instrument are shown in Figs. 6 and 7. The various parts will be easily recognised from the description already given. The photographs

show the simple arrangement which has been found satisfactory for operating the wedges. These simply lie against the faces of the slit mounts and are suspended by threads which pass over pulleys on a support clamped to the collimators. These threads are wound round drums attached to operating handles convenient to the right hand of the operator. His left hand operates the wave-length drum. A suitable support to carry filters of the standard 2-inch square type is fitted in front of both collimators. That in front of the extra collimator carries the filters used to produce

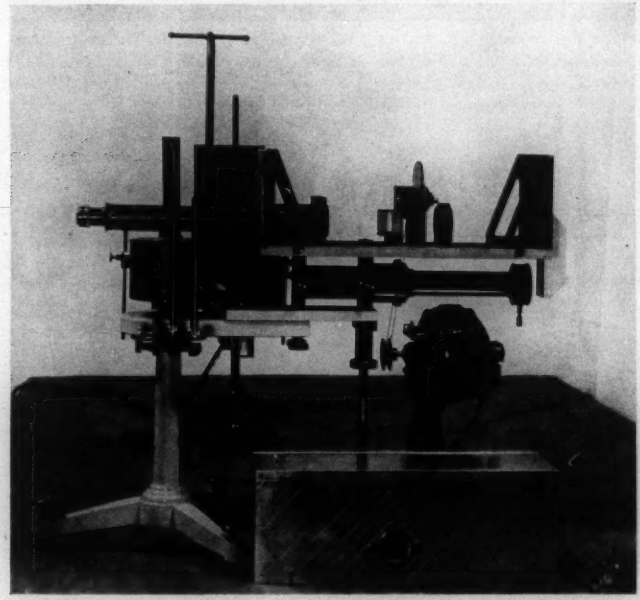

Fig. 7

the fixed colours. That in front of the spectroscopic collimator may be used to hold suitable filters for eliminating scattered light, thereby ensuring that the spectral component is pure.

No mention has yet been made of the means whereby a third fixed colour (*g*, Fig. 3) may be added to the colour under test when it is desired to employ the procedure described in connexion with that figure. A separate piece of apparatus of a very simple character is employed for this purpose. Its use will be readily

followed from Fig. 8, which shows a diagrammatic plan of part of the colorimeter in relation to the standard illuminating arrangements referred to in the preceding footnote. A filter holder *H*, wide enough to contain about three standard filters, is mounted on a suitable stand. A thin sheet of colourless optical glass *G* is mounted in a light frame which is hinged to the filter holder. The glass plate may be brought out to an inclination of 45° with the plane of the filter holder, or may be folded flat against it. The stand is placed so that when the glass is in the former position it crosses the beam from the specimen to the colorimeter as shown in the figure. When the glass is closed up against the filter holder, however, it is completely out of the beam.

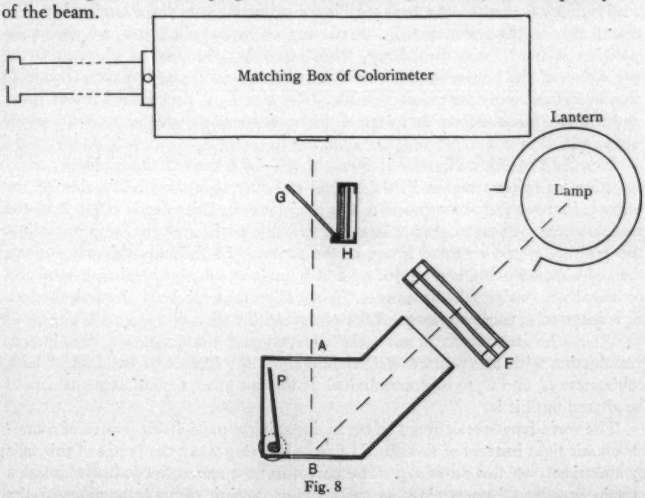

Fig. 8

A suitable green filter is placed in the holder and also a sheet of Ilford diffusing medium, or *thin* opal, which should be placed on the side of the filter remote from *G*. The lantern used to contain the standard lamp should have an aperture in it which permits light from the lamp to illuminate the diffusing screen. In these circumstances some green light, reflected at the plate *G*, will be superposed on the colour under test. In order, if need be, to reduce the amount of this added colour, an additional *neutral* filter may be inserted in the holder. One or two members of the Wratten neutral series, densities ·3, ·6, and 1·0, say, will meet all practical requirements. When the stand carrying this auxiliary piece of apparatus has once been set in position, the added colour can be thrown in or out at will by moving the hinged plate *G*. A stop is provided to facilitate returning the plate to the same position every time.

Practical Adjustments

To make the necessary adjustments of this apparatus, the crown of lenses should be stationary, in such a position that the image of the slit at E (Fig. 4) is approximately central. The slit of C_2 should be illuminated by any convenient means and the plate G adjusted until the image of the slit of C_3 falls symmetrically on the slit of C_3. The progress of the adjustment is easily observed if an eyepiece is employed at E. The slit of C_2 should always be wider than that of C_3 in order that the latter may be completely filled.

Having adjusted G, the next step is tne adjustment of the illumination. We attend first to the collimator C_2. Removing all lenses and filters, we adjust the position of the "Pointolite" lamp, while observing the field of view, until the projection of the luminous plate which can be seen in the appropriate section of the field is as nearly central as possible. The lens L_2 is then placed in the beam and adjusted so as to form an image of the luminous plate which covers the whole of the slit.

The light to this collimator is then cut off—by means of the wedge w_2 or by inserting an opaque screen in the beam—in order that the illumination of the other collimator may be adjusted. The alignment in this case is effected in the same way except that we do not interfere with the position of the lamp but adjust the position of the reflected image of the source. To facilitate this adjustment, the right-angled prism is mounted on a little table attached to the lamp stand and provided with universal adjustments. When alignment has been effected, the lens L_1 is inserted to focus an image of the source on the slit.

When the instrument is correctly adjusted and both slits are illuminated, examination with an eyepiece at E should show the images of the slits of both collimators C_1 and C_2 to be coterminous. If this is not so, the adjustments should be altered until it is.

The wave-length scale must be set by inserting some suitable source of monochromatic light in front of the slit of C_1 and observing when the image of this falls symmetrically on the slit of C_3. The curvature of spectral lines always causes a certain amount of uncertainty as to the exact setting of an instrument of the monochromator type. It is desirable therefore to check the adjustment by an actual colour match. This can be done by measuring, by colour match, the wave-length of yellow sodium light, obtained from a good "sodium flame" and purified by passing through a yellow filter.

The axis of the observing collimator C_4 is adjusted, by means of three screws* in a collar through which it passes loosely, so that as large an opening of the iris diaphragm as possible may be completely covered by the patch of light into which the slit image is spread when the moving lenses are in operation.

* Only one is shown in Fig. 4, the others are at 120 degrees from this.

Standard Filters

A suitable filter for use as a standard red is the "Spectrum Red" of the Ilford spectrum series of filters. This gives a colour indistinguishable from the terminal red of the spectrum. For colours near the other end of the spectrum I have found both the spectrum violet and spectrum blue of the same series useful, particularly the blue, which is near enough the end of the spectrum for most purposes. These filters give colours sensibly identical with those of the spectrum at wave-lengths ·447μ and ·473μ respectively. Different specimens of these filters are not identical in hue, and this has to be determined for the actual filters used. This is easily done by measuring the colour of the filters with the instrument, using of course an illuminant of similar colour to that of the "Pointolite" lamp. As they give fully saturated colours, the measurement merely consists in setting the wave-length drum. As these colours are, however, to be employed as fundamental constants of the instrument, the values adopted for each should be the statistical mean of a large number of observations, and should be checked from time to time.

A suitable green filter which gives a very definite colour of full saturation is No. 62 of the Wratten series (Eastman Kodak). The wave-length of the spectrum colour to which this is equivalent is easily determined by direct measurement when this light alone is in the field of view. It is approximately ·537μ for the specimens at this Laboratory.

Apart from these fixed colours suitable for a general method of colorimetry, we can clearly utilise for special experimental purposes any fixed colour obtainable by means of a filter. There are filters available which enable us to get a number of fixed points throughout the spectrum and mixtures of any of these colours with other monochromatic colours can be made with the instrument.

Utility of the Vector Method

The instrument just described was designed primarily as a means of measuring by a simple and accurate procedure the *quality* of colours, particularly for work involving colours of high spectral saturation, which are the least convenient for dealing with on trichromatic colorimeters. That it can deal with all colours we have seen, but it is not as a universal instrument that its chief advantages are claimed for it. There is one practical respect, however, in which the method is, apparently, unique. It is the only method, as far as I am aware, in which the quantitative relationships of the different colours entering into the colour matches are not involved in the measurement of colour quality. In a trichromatic colorimeter, for instance, we have to assume that the scale units of the different primaries have the same relative values when making the colour match as they had when matching the standard white. This means that the illuminant employed for the primaries must be operated under standard conditions, and that the design and adjustment of the apparatus must be such that the ocular pupil is uniformly illuminated all

over by the various pencils of light entering the eye, to prevent changes in the scale values due to slight differences in the position of the observer's head. Those familiar with colorimetric apparatus know how difficult it is to obtain a colour match which is identical all over the field of view and is not appreciably affected by changes in position of the head.

With the vector method, however, such difficulties do not endanger the validity of the result. No quantitative relationship is involved, so the "Pointolite" lamp need not be run under any special conditions; no scales are required, other than the wave-length scale, and uniformity of colour all over the field, while desirable, is not essential. If the field is matched *at any point* it proves that the correct monochromatic constituent is being used, which is all the information required.

The method is easy and rapid in practice when once the observer has acquired that initial experience of the effect of the various controls which is required in colorimetric work by any method. Even when used in those cases which require the procedure of Fig. 3, the additional trouble involved is very much less than might be thought from the description of the process. The ease with which it is carried out, together with its freedom from the difficulties referred to, renders it one of the most fool-proof methods which are available for measurements of colour quality. It is pre-eminently suitable for measurements on the colour of illuminants, or other work in which the luminosity of the colour is not required as part of the colorimetric information.

The measurement of luminosity, where it is necessary to take this into account, as in differentiating between colours of the same quality but different luminosities, must, in the author's view, be regarded as a quite separate problem, being essentially photometric rather than colorimetric. It is in many cases best carried out with other apparatus than is used for the purely colorimetric measurements. I shall have more to say on the general problem of measurement of transmission or reflection factors of coloured materials in a later paper. In small laboratories, however, it is usually imperative to get all the information possible with a limited amount of apparatus, and it is desirable for this reason that it should be possible to evaluate the fractional luminosity of the light transmitted or reflected by materials on any colorimeter, even though the method employed is not that which would be selected if apparatus were available for alternative methods.

To make such measurements possible on the vector colorimeter it is only necessary to fit a scale to the photometric wedge w_2 (Fig. 4), and to calibrate this for the light transmitted by the "spectrum red" filter r (Figs. 1, 2, and 3), so that the relative luminosity of r for different positions of the wedge may be known.

A luminosity scale for this fixed colour alone enables us to measure the luminosity fraction for any coloured material, as this can be deduced from the relative amounts of the fixed red which are required in a match of the colour under test and of the standard white respectively. I shall merely describe the method here; the basis of it will be easily understood if the reader has perused the paper on colour calculations to which previous reference has been made.

The magnesium oxide screen in the illuminating device pictured in Fig. 8 is

illuminated with a standard white light*. A match is made in the field of the colorimeter by mixing the fixed red with suitable monochromatic light. The position of the wedge w_2 is noted. The specimen is then placed in the path of the reflected light if it is transparent, or substituted for the magnesia screen if it is opaque, no alteration being made in the position of the standard lamp. The position of the colour on the chart is found by the procedure described earlier, note being made of the wedge reading for the match involving the fixed red. From these wedge readings the relative luminosity of r in the match of the colour as compared with its luminosity in the match of white becomes known.

Let us suppose that C (Fig. 2) is the colour in question. The fraction of its luminosity which is due to r is

$$\frac{L_r \times CH}{L_r \times CH + L_H \times rC},$$

where L_r and L_H are the relative luminosities of one trichromatic unit of each of the colours r and H.

Thus in any mixture of r with H which has the colour quality denoted by C, the ratio of the luminosity of the mixture to that of the red constituent alone is

$$1 + \frac{L_r}{L_H}\frac{CH}{rC} = \rho, \text{ say.}$$

The necessary data for calculating ρ will be found in the paper on colour calculations, where the luminosity coefficients of trichromatic units of the spectrum colours are tabulated on the basis of the Koenig-Ives colour chart data. L_r, which enters into every such calculation, is ·550.

The spectral colour which must be mixed with r to match white is, according to the Koenig-Ives chart, the colour at wave-length ·492μ. By a calculation of similar character to the above we find that the mixture of r with light of wave-length ·492μ which matches white is 1·95 times as luminous as the red constituent alone.

Hence if the luminosity of r when matching the test colour is α times its luminosity when matching the standard white, the ratio of the luminosity of the test colour to the white is

$$\alpha \frac{\rho}{1{\cdot}95}.$$

It will be evident that the method becomes of low precision for colours in which the proportion of r is low, and for fully saturated colours (with the exceptions mentioned below) it is quite inapplicable, as the proportion of the fixed colour required to match these is zero. The exceptions are spectral colours from the red end of the spectrum to about the yellow green. Although such colours may be matched by monochromatic light of the proper wave-length (and their hue must be so determined), it is nevertheless possible, on account of the approximate straightness of the spectrum locus, to make a very good match of any colour in this range by mixing r with any spectrum colour between that under test and the bend in the spectrum locus which begins to become pronounced about ·55μ.

* See *Trans. Opt. Soc.* 27 (1925–26) 106, Appendix I (p. 122).

Thus, in order to get a luminosity measurement on a saturated colour of, say, ·60μ, we may match it quite well enough with a mixture of r and monochromatic light of ·56μ or ·57μ to enable the procedure already described to be applied. For colours of full saturation of other hues than these in the region considered we cannot obtain any match containing r.

For such colours we must employ a modified procedure, using the device already described (Fig. 8) for adding a known colour to that under test. For the present purpose we replace the green filter by a purple one, in order to obtain a fixed colour, such as p (Fig. 9), as far removed from the spectrum locus as conveniently possible.

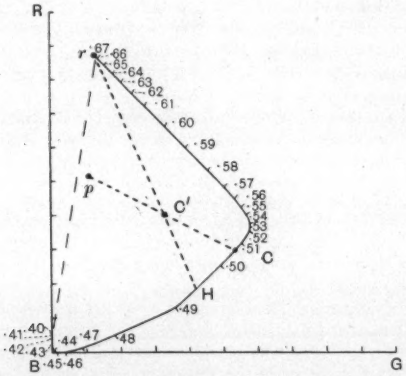

Fig. 9

Suppose C (Fig. 9) is the saturated colour for which we wish to measure the luminosity fraction. We add to it sufficient of the purple p to obtain a colour C' well inside the colour field. We then match C' with a mixture of r and a suitable monochromatic colour H. This, in conjunction with the fact that C' must lie on pC, determines the position of C' on the chart. We can now calculate the relative luminosity of any quantity of C' to that of the amount of r in it in the ordinary way, and, by comparing the r readings for this match and for the standard white, we can deduce the ratio of the luminosity of the amount of C' which was present in the matching field to that of the standard white, just as though C' and not C were the colour under test.

The luminosity of the amount of C' produced by the addition of the purple to the test colour is the sum of the luminosities of the test colour and the purple. The fraction of the total luminosity due to the initial colour is

$$\frac{pC' \times L_c}{C'C \times L_p + pC' \times L_c} = \gamma, \text{ say,}$$

where L_c and L_p have their usual significance as the relative luminosities of one trichromatic unit of C and p respectively. L_c is obtained from the data given in the paper on colour calculations, while L_p is calculated from the luminosity coefficients of the primaries, as explained in the same paper.

If then we find that the ratio of the luminosity of C' to the standard white is β, the ratio of the luminosity of the test colour C to white, which is the required information, is $\beta\gamma$.

Any good purple filter, say Wratten No. 35, will serve the purpose. The position of p on the chart has to be found, of course, by measurement and should be determined with the care desirable in fixing a standard for subsequent use.

Thus we can, if desired, obtain a quantitative measurement of reflectivity or transmission factors in terms of the quantities of r alone which enter into various matches. The method is fairly convenient, but is not, of course, a direct measurement of luminosity, inasmuch as the final value obtained depends on the accuracy of the luminosity data of the chart. It is an indirect method, similar in principle to the method of deducing luminosities from measurements on a trichromatic colorimeter, and would give correct results if the data were accurate. Whether the errors liable to arise from inaccuracy of data are likely to be greater with this method than with the trichromatic method I have not enough experience of luminosity determinations by both methods to say.

As pointed out earlier, the method is suggested, not as one specially designed for dealing with luminosity measurements, but as a possible way in which quantitative information can be obtained where for practical reasons it may be desired to obtain it on an instrument primarily intended for colorimetric work properly so called. It is for the latter purpose that the vector colorimeter is advanced as an important addition to available equipment: extensive experience may show it to be either more or less suitable than other instruments for auxiliary purposes such as have just been discussed.

The instrument illustrated in the paper was constructed in the Optics Workshop of the National Physical Laboratory, using a standard Hilger constant deviation spectrometer as a basis.

DISCUSSION

Mr J. W. Perry (communicated): In my opinion Mr Guild has by this and the preceding papers considerably advanced the status of the physical problems relating to colour. In connexion with some apparently very general remarks there must, however, be borne in mind the restrictions which the author has imposed upon the related discussions.

The vector method of colorimetry proposed by Mr Guild is noteworthy from the point of view of the greater generality of the underlying principles. It may, however, be asked whether he does not go beyond the evidence. It is an essential

postulate in the system employed that the quality of a colour be unaffected by variation of its photometric quantity. This is undoubtedly to physicists a very desirable property. In regard to luminosity of a complex radiation, that the whole is equal to the sum of its parts has, as is known, been established already by experiment: but, as far as I am aware, it is not yet securely founded as an experimental fact that the specification of colour quality is possible by use of two variables only, and is independent of luminosity. Only when it is definitely known that this applies within the limits of illumination occurring in practice will such a detached colorimetric system possess ultimate signification. In the case of pre-existing instruments, considered only from the point of view of basic method, an absolute chromatic analysis is aimed at, in which case such considerations as the above are irrelevant.

This, nevertheless, does not detract from the inherent elegance and undoubted ultimate utility of the new method. Even were it granted that such controverted secondary variations were existent, but of secondary magnitude, it is probable that the practical judgment would favour that method and instrument which were least exposed to objections implicating the ultimate precision.

Mr Guild (communicated): The vector method does not really differ as regards its underlying postulates from any other fundamental method of colorimetry. The apparent difference is due to the fact that the method only professes to apply to one of the two separable characteristics of a visual sensation, namely colour quality, and therefore calls attention to the fact that this is being treated independently of luminosity, whereas on instruments employing other methods it is usually possible to make determinations of *relative* luminosity, where these are relevant, as well, and the essential independence of the two aspects of the sensation is not so obvious; but it is there to precisely the same extent.

I shall deal presently with the question of this independence, but first let me show the identity of this and other methods as regards the conditions in which colour quality is measured. In any colorimeter whatever we match the colour at some arbitrary luminosity at which it happens to appear in the field of view. Usually no particular luminosity is specified, it being known from experience that the quality of the colour will *measure* the same over a very wide range of luminosity. If for any particular purpose a certain intensity of illumination of the colour under test is specified, this can be employed when using the vector method just as easily as when using any other. Thus in no method of colorimetry is it usual, or in general necessary, to take account of the absolute luminosity of the visual sensation. The *fractional* luminosity of a coloured material, i.e. the fraction of the incident light which is reflected or transmitted by the material, must not be confused with the luminosity of the sensation. It is simply a photometric ratio and does not define the luminosity level at which the observations are made. While fractional luminosity is just as important as colour quality in defining the appearance of an object relative to the appearance of other objects, it is not an essential or relevant factor in a colorimetric measurement or specification, properly so called. It is a

separate property which can be measured—and is possibly best measured—by methods not involving colour matches at all.

Thus, if one analyses the information actually obtained in colorimetric measurements by any method of which I am aware, it will be found that the assumption of independence of colour quality and luminosity is common to all methods. I have said assumption, but it is not so much assumption as a matter of definition. We have to bear in mind that in colorimetry we are never concerned with the actual appearance of colours but only with the *identity* of the appearance of a colour with that of some definable standard. Thus we are not concerned with the question whether the quality of a colour in some absolute sense does or does not remain constant with changes of luminosity. I don't know whether any absolute definition of colour quality could be formulated—if so it would have to be in terms of the possible types of activity of some mental process or processes—but in any case we cannot test by physical measurements any visual property, so defined, as our only possible tests consist of comparisons of external stimuli which produce identity of sensation, and are essentially relative.

For purposes of measurement and definition we have to separate the complex visual sensation into such elements as are most conveniently compared with standards. Now it is established to a high degree of experimental accuracy that luminosity can be regarded as independent of the other elements of visual sensation in the sense that a metrical scale of luminosity can be devised, and realised in practice, which has the necessary properties of a metrical scale that things which measure equal to the same thing will also measure equal to one another, and that two quantities which measure respectively x and y units will, if added together, measure as $x + y$. The fact that such a scale is realisable makes it convenient to take luminosity as one of the separate elements for measurement purposes. The elements of the sensation which remain go to make up what, for colorimetric purposes, I have termed colour quality. In fact we define this term by saying that two colours are of the same quality which can be made to look alike by merely altering the luminosity of one of them. Thus suppose we have two pieces of coloured paper which appear quite dissimilar when subject to the same illumination. If we arrange matters so that the illumination on one of them can be independently varied, without altering its energy distribution, as, for example, by moving a lamp nearer or farther away, we shall find one of two things. Either we cannot make the papers look alike with any adjustment of the lamp, or else, for a particular adjustment, they may be made to look alike. In the latter case these papers, though differing in fractional luminosity and, therefore, in colour as this term is generally used in reference to objects, are identical in colour quality as defined above. They would have the same hue and saturation, and would be represented by the same point on the colour chart. It is colour quality which is always meant when the term colour is applied to illuminants, as when referring to the colour of the light from an acetylene flame, a mercury vapour lamp, etc., in which cases the candle-power of the lamps, or their distance from the screen on which the light is being received, is never taken into account. Colour quality,

then, in the sense used here, is *by definition* independent of luminosity. It remains of course to show that the entity so defined is practically measurable. The proof of this lies in the experimental fact that colour matches hold over a very large range of luminosity, and that the same values are obtained in terms of standard data, that is to say, the same position is found for a colour on the colour chart, at whatever luminosity, within very wide limits, the measurements are made.

What precisely are the safe limits of luminosity for colorimetry I cannot say, but constancy of the colour-matching properties of the eye can certainly be safely assumed over a wider range of luminosity than constancy of the relative luminosity function.

It will be evident from the foregoing that the vector method introduces no new principles of colour vision. The novelty is one of method only and not of theory. The method is merely a new application of principles which are fundamental to the whole structure of modern colorimetric science. That these principles are at least nearly true may be regarded as already established. The point, if any, at which they fail can only be determined when the technique of colorimetry has been brought to a sufficiently high level to reveal such ultimate discrepancies as may exist. At present there is no evidence that the principles fail at all.

REVIEW

Glass Research Association Bulletin, No. 14: A Review of the Work of the Association, 1920–1925. Pp. 106; 3 Figs., 3 Plates. (London: Glass Research Association (in Voluntary Liquidation), WILFRID C. SMITH, A.C.A., Liquidator, 90 Cannon Street, E.C. 4, 1925.)

This number of the *Glass Research Association Bulletin* contains a review of the research work carried out by the Association during the five years of its existence. At the time when the Association was established, the state of the existing scientific knowledge of glass and its properties, as well as the condition of the glass industry itself, was such as to afford wide scope for research. The report now before us shows that the work has been developed on broad lines, several of the investigations having been of a fundamental nature relating to the properties of glass, while others have been directed towards providing means of improving and cheapening production. Investigations having for their aim the increase of our knowledge relating to the processes involved in the melting of glass and to the improvement of refractories for use in glass making have received considerable attention. Work on optical glass, as such, has not been undertaken as it was considered that this branch of the subject was receiving sufficient attention from the British Scientific Instrument Research Association.

The researches have been conducted at the Association's Laboratories and at the National Physical Laboratory and the Department of Glass Technology in the University of Sheffield. In addition, the Association has co-operated with the British Refractories Research Association, the British Cast Iron Research Association, and the Industrial Fatigue Research Board in regard to investigations of importance to the glass industry and cognate to the work on which these bodies were engaged.

Experimental work on glass is of a particularly difficult and lengthy character, and continuous and co-ordinated research work over a much longer period than that under review would be required to lay a sound foundation for a science of glass technology, and a still longer period to secure the adequate application of its results to the industry. Nevertheless, the results enumerated in the report indicate that important advances in both directions have been made. Suggested programmes are given of further researches on glass and on refractories which, in the opinion of the Research Committees of the Association, are of prime importance and ought to be undertaken immediately. It seems desirable that these suggestions should be brought before the notice of a wider circle of research workers than that in which the *Bulletin* circulates and, as the report is not offered for sale, reference may usefully be made here to some of the suggested items of research.

It is considered that fundamental researches on glass will naturally be directed to the study of the constitution and structure of vitreous substances both in the molten and solid condition, and the following investigations are suggested:

(1) The measurement of physical constants, such as (*a*) viscosity, (*b*) electrical properties, (*c*) surface tension (probably in connection with (*a*)).
(2) Behaviour of glass under electrolysis over a wide range of temperatures.
(3) Study of X-ray diffraction patterns obtained both from glass and from glass making materials at various stages during the melting process.
(4) Thermal study of glass, particularly by means of heating and cooling curves under various conditions.

If the above investigations are to be successfully prosecuted, certain conditions are essential. These relate mainly to the preparation of samples of glass of adequate purity and homogeneity. This is in itself a problem of very great difficulty and will no doubt require a considerable amount of experimental research. The chief factors affecting the purity of the glass are the purity of the material and the containers, and the methods of mixing and melting. The difficulty of satisfying this condition as to purity and homogeneity will probably lead to the adoption of experimental methods involving the use of only small quantities of glass, as otherwise the cost and difficulty of preparation may be too great. On the other hand, some of the difficulties are very considerably enhanced if only small quantities are used, and it may be necessary to work with samples of homogeneous glass obtained from large meltings prepared industrially for such purposes as those of optical glass.

Investigations along the lines indicated above are required in order to lay a rational foundation to the application of science and scientific methods to glass production generally. Among the investigations directed towards more immediately practical ends, the following are suggested as the most important:

(1) The mechanical or "strength" properties of glass in relation to mode of treatment and composition.

(2) Thermal properties of glass (conductivity, expansion, emissivity) at all temperatures up to the melting temperature.

(3) The solubility of gases in glass at various temperatures and pressures, their mode of evolution and absorption, and their effect on the physical properties of the glass.

The above programmes cover a very wide range and would furnish work for a considerable number of highly skilled investigators provided with the best possible equipment for a period of years. The need for research on glass is still urgent, and it is to be hoped that such work will be prosecuted continuously and as rapidly as possible.